

إجابات كتاب التمارين

التكامل بالتعويض

أجد كلاً من التكاملات الآتية:

$$\int (x^2 + 3) dx \quad (1)$$

$$\begin{aligned} x^2 + 3 dx u = x^2 + 3 \Rightarrow du dx = 2x \Rightarrow dx = \frac{du}{2x} \\ \int (x^2 + 3) dx = \int x u^2 \frac{du}{2x} = \frac{1}{2} \int u^2 du \\ = \frac{1}{2} \cdot \frac{u^3}{3} + C = \frac{1}{6} (x^2 + 3)^3 + C \end{aligned}$$

$$\int (x^4 e^{x^5} + 2) dx \quad (2)$$

$$\begin{aligned} x^4 e^{x^5} + 2 dx u = x^5 + 2 \Rightarrow du dx = 5x^4 \Rightarrow dx = \frac{du}{5x^4} \\ \int (x^4 e^{x^5} + 2) dx = \int x^4 e^u \frac{du}{5x^4} + \int 2 dx \\ = \frac{1}{5} \int e^u du + 2x + C = \frac{1}{5} e^{x^5} + 2x + C \end{aligned}$$

$$\int (x+1)(x^2+2x+5)^4 dx \quad (3)$$

$$\begin{aligned} (x+1)(x^2+2x+5)^4 dx u = x^2 + 2x + 5 \Rightarrow du dx = 2x + 2 \Rightarrow dx = \frac{du}{2x+2} \\ \int (x+1)(x^2+2x+5)^4 dx = \int (x+1) u^4 \frac{du}{2x+2} \\ = \frac{1}{2} \int u^4 du = \frac{1}{2} \cdot \frac{u^5}{5} + C = \frac{1}{10} (x^2 + 2x + 5)^5 + C \end{aligned}$$

$$\int (x)^3 x dx \quad (4)$$

$$\begin{aligned} (x)^3 x dx = \int u^3 x du = \int 0 u^3 du = x \Rightarrow du dx = 1/x \Rightarrow dx = x du \\ \int (x)^3 x dx = \int u^3 du = \frac{1}{4} u^4 + C = \frac{1}{4} (\ln x)^4 + C \end{aligned}$$

$$\int (x dx)^5 \sin^4 \cos^3 x \quad (5)$$

$$\begin{aligned} (x dx)^5 \sin^4 \cos^3 x = \int \cos^3 x \sin^4 x dx \\ \cos^3 x = \cos x \sin^2 x = \cos x (1 - \cos^2 x) \\ \int \cos x (1 - \cos^2 x) \sin^4 x dx = \int (1 - u^2) u^4 du \\ = \int (u^4 - u^6) du = \frac{1}{5} u^5 - \frac{1}{7} u^7 + C \\ = \frac{1}{5} \sin^5 x - \frac{1}{7} \sin^7 x + C \end{aligned}$$

$$\int (x dx)^6 \cos^3 \sin^4 x \quad (6)$$

$$\begin{aligned} (x dx)^6 \cos^3 \sin^4 x = \int \cos^3 x \sin^4 x dx \\ \cos^3 x = \cos x \sin^2 x = \cos x (1 - \cos^2 x) \\ \int \cos x (1 - \cos^2 x) \sin^4 x dx = \int (1 - u^2) u^4 du \\ = \frac{1}{5} u^5 - \frac{1}{7} u^7 + C \\ = \frac{1}{5} \sin^5 x - \frac{1}{7} \sin^7 x + C \end{aligned}$$

أجد قيمة كل من التكاملات الآتية:

$$\int_1^2 (12x^2(x^3+1)^2) dx \quad (7)$$

$$\begin{aligned} 12x^2(x^3+1)^2 dx &= u^2 \Rightarrow du = 3x^2 dx \Rightarrow dx = \frac{du}{3x^2} \\ \int_1^2 12x^2(x^3+1)^2 dx &= \int_2^9 2u^2 \frac{du}{3} = \frac{2}{3} \int_2^9 u^2 du = \frac{2}{3} \left[\frac{1}{3} u^3 \right]_2^9 = \frac{2}{9} (9^3 - 2^3) = \frac{2}{9} (729 - 8) = \frac{2}{9} \cdot 721 = \frac{1442}{9} \end{aligned}$$

$$\int_0^1 (1x^3x^2+2) dx \quad (8)$$

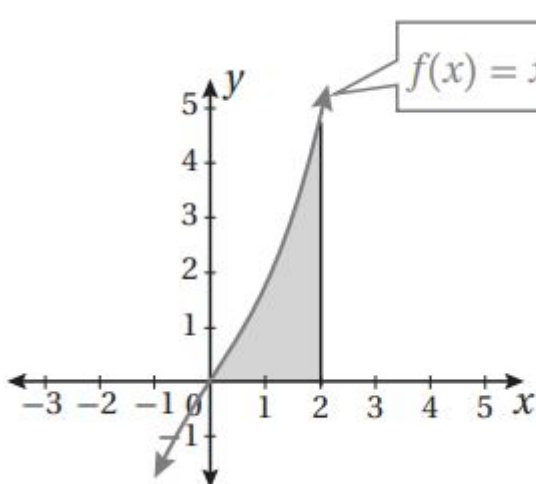
$$\begin{aligned} 1x^3x^2+2 dx &= u^2 \Rightarrow du = 6x dx \Rightarrow dx = \frac{du}{6x} \\ \int_0^1 (1x^3x^2+2) dx &= \int_2^5 \frac{1}{6} u^2 \frac{du}{6x} = \frac{1}{36} \int_2^5 u^2 du = \frac{1}{36} \left[\frac{1}{3} u^3 \right]_2^5 = \frac{1}{108} (5^3 - 2^3) = \frac{1}{108} (125 - 8) = \frac{117}{108} = \frac{13}{12} \end{aligned}$$

$$\int_1^e (x)^{2x} dx \quad (9)$$

$$\begin{aligned} x^{2x} dx &= u^2 \Rightarrow du = 2x dx \Rightarrow dx = \frac{du}{2x} \\ \int_1^e (x)^{2x} dx &= \int_2^{e^2} \frac{1}{2} u^2 \frac{du}{2x} = \frac{1}{4} \int_2^{e^2} u^2 du = \frac{1}{4} \left[\frac{1}{3} u^3 \right]_2^{e^2} = \frac{1}{12} (e^6 - 8) \end{aligned}$$

$$\int_0^1 (x+1)(x^2+2x)^5 dx \quad (10)$$

$$\begin{aligned} (x+1)(x^2+2x)^5 dx &= u^5 \Rightarrow du = (2x+2) dx \Rightarrow dx = \frac{du}{2(x+1)} \\ \int_0^1 (x+1)(x^2+2x)^5 dx &= \int_0^3 \frac{1}{2} u^5 du = \frac{1}{2} \left[\frac{1}{6} u^6 \right]_0^3 = \frac{1}{12} (3^6 - 0) = \frac{1}{12} \cdot 729 = \frac{243}{4} \end{aligned}$$



(11) أجد مساحة المنطقة المظللة في التمثيل البياني المجاور.

$$\begin{aligned} A &= \int_0^2 x \sqrt{x^2+2} dx \\ x \sqrt{x^2+2} dx &= u^2 \Rightarrow du = 2x dx \Rightarrow dx = \frac{du}{2x} \\ \int_0^2 x \sqrt{x^2+2} dx &= \int_2^6 \frac{1}{2} u^2 \frac{du}{2x} = \frac{1}{4} \int_2^6 u^2 du = \frac{1}{4} \left[\frac{1}{3} u^3 \right]_2^6 = \frac{1}{12} (6^3 - 2^3) = \frac{1}{12} (216 - 8) = \frac{208}{12} = \frac{26}{3} \end{aligned}$$

(12) الإيراد الحدي: يمثل الاقتران: $R'(x) = 50 + 3.5xe^{-0.1x^2}$ الإيراد الحدي (بالدينار) لكل قطعة تباع من إنتاج إحدى الشركات، حيث x عدد القطع المباعة، و $R(x)$ إيراد بيع x قطعة بالدينار. أجد اقتران الإيراد $R(x)$ ، علماً بأن $R(0) = 0$.

$$R(x) = \int (50 + 3.5xe^{-0.1x^2}) dx = \int 50 dx + \int 3.5xe^{-0.1x^2} dx = 50x + \int 3.5xe^{-0.1x^2} dx$$

$$u = -0.1x^2 \Rightarrow \frac{du}{dx} = -0.2x \Rightarrow dx = \frac{du}{-0.2x}$$

$$\int 3.5xe^{-0.1x^2} dx = \int 3.5x e^u \frac{du}{-0.2x} = -17.5 \int e^u du = -17.5e^u = -17.5e^{-0.1x^2} + C$$

$$R(0) = 0 \Rightarrow 0 - 17.5 + C = 0 \Rightarrow C = 17.5$$

يمثل الاقتران $f'(x)$ في كل مما يأتي ميل المماس لمنحنى الاقتران $f(x)$ المار بالنقطة المعطاة، أستعمل المعلومات المعطاة لإيجاد قاعدة الاقتران $f(x)$:

(13) $f'(x) = 2x(4x^2 - 10)^2; (2, 10)$

$$f(x) = \int 2x(4x^2 - 10)^2 dx$$

$$u = 4x^2 - 10 \Rightarrow \frac{du}{dx} = 8x \Rightarrow dx = \frac{du}{8x}$$

$$\int 2x(4x^2 - 10)^2 dx = \int 2x u^2 \frac{du}{8x} = \frac{1}{4} \int u^2 du = \frac{1}{12} u^3 + C$$

$$f(2) = 10 \Rightarrow \frac{1}{12} (4 \cdot 2^2 - 10)^3 + C = 10 \Rightarrow \frac{1}{12} (16 - 10)^3 + C = 10 \Rightarrow \frac{1}{12} (6)^3 + C = 10 \Rightarrow 3 + C = 10 \Rightarrow C = 7$$

$$f(x) = \frac{1}{12} (4x^2 - 10)^3 + 7$$

(14) $f'(x) = x^2 e^{-0.2x^3}; (0, 32)$

$$f(x) = \int x^2 e^{-0.2x^3} dx$$

$$u = -0.2x^3 \Rightarrow \frac{du}{dx} = -0.6x^2 \Rightarrow dx = \frac{du}{-0.6x^2}$$

$$\int x^2 e^{-0.2x^3} dx = \int x^2 e^u \frac{du}{-0.6x^2} = -\frac{1}{0.6} \int e^u du = -\frac{1}{0.6} e^u + C = -\frac{5}{3} e^{-0.2x^3} + C$$

$$f(0) = 32 \Rightarrow -\frac{5}{3} e^{-0.2 \cdot 0^3} + C = 32 \Rightarrow -\frac{5}{3} e^0 + C = 32 \Rightarrow -\frac{5}{3} + C = 32 \Rightarrow C = 32 + \frac{5}{3} = \frac{96 + 5}{3} = \frac{101}{3}$$

$$f(x) = -\frac{5}{3} e^{-0.2x^3} + \frac{101}{3}$$

(15) يتحرك جسيم في مسار مستقيم، وتعطى سرعته المتجهة بالاقتران:

$v(t) = t^2 + 1$ ، حيث t الزمن بالثواني، و v سرعته المتجهة بالمتري لكل ثانية. إذا بدأ الجسيم حركته من نقطة الأصل، فأجد موقعه بعد t ثانية من بدء الحركة.

$$s(t) = \int t^2 + 1 dt = \frac{1}{3} t^3 + t + C$$

$$\frac{ds}{dt} = 2t \Rightarrow dt = \frac{ds}{2t}$$

$$\int t^2 + 1 dt = \int \frac{1}{2} (u - 1)^2 + 1 \frac{du}{2} = \frac{1}{4} \int (u - 1)^2 + 2 du = \frac{1}{4} \left(\frac{1}{3} (u - 1)^3 + 2(u - 1) \right) + C$$

$$s(t) = \frac{1}{4} \left(\frac{1}{3} (2t + 1 - 1)^3 + 2(2t + 1 - 1) \right) + C = \frac{1}{4} \left(\frac{1}{3} (2t)^3 + 4t \right) + C = \frac{1}{6} (2t)^3 + t + C = \frac{2}{3} t^3 + t + C$$

$$s(0) = 0 \Rightarrow \frac{2}{3} (0)^3 + 0 + C = 0 \Rightarrow C = 0$$

$$s(t) = \frac{2}{3} t^3 + t$$