إجابات أسئلة الدرس

التكامل بالتعويض

١) اكتب التعويض المناسب لإيجاد قيمة كل تكامل من التكاملات الآتية:

أ) $\int (1 - 2\text{س})(\text{س} - \text{س}^2)^4 \, \text{دس}$

ب) $\int 6\text{س}^2 \sqrt[5]{(2\text{س}^3 - 2)^2} \, \text{دس}$

جـ) $\int (2\text{س} - 3\text{س}^2) \text{قا}^2(\text{س}^2 - \text{س}^3) \, \text{دس}$

د) $\int \frac{3\text{س} - 9}{(\text{س}^2 - 6\text{س})^2} \, \text{دس}$

الحل

أ) $\int (1 - 2\text{س})(\text{س} - \text{س}^2)^4 \, \text{دس}$.

$\text{ص} = \text{س} - \text{س}^2 \Leftarrow \frac{\text{دص}}{\text{دس}} = 1 - 2\text{س} \Leftarrow \text{دس} = \frac{\text{دص}}{1 - 2\text{س}}$

$\int \cancel{(1 - 2\text{س})} \, \text{ص}^4 \, \frac{\text{دص}}{\cancel{1 - 2\text{س}}} = \int \text{ص}^4 \, \text{دص}$

$= \frac{\text{ص}^5}{5} + \text{جـ} = \frac{(\text{س} - \text{س}^2)^5}{5} + \text{جـ}$

ب) $\int 6\text{س}^2 \sqrt[5]{(2\text{س}^3 - 2)^2} \, \text{دس}$

$\text{ص} = 2\text{س}^3 - 2 \Leftarrow \frac{\text{دص}}{\text{دس}} = 6\text{س}^2 \Leftarrow \text{دس} = \frac{\text{دص}}{6\text{س}^2}$

$\int \cancel{6\text{س}^2} \sqrt[5]{\text{ص}^2} \, \frac{\text{دص}}{\cancel{6\text{س}^2}} = \int \text{ص}^{\frac{2}{5}} \, \text{دص}$

$$\frac{\text{ص}^{\frac{2}{5}+1}}{\frac{2}{5}+1} + \text{جـ} = \frac{\text{ص}^{\frac{7}{5}}}{\frac{7}{5}} + \text{جـ}$$

$$= \frac{5}{7}\sqrt[5]{\text{ص}^{7}} + \text{جـ}$$

$$= \frac{5}{7}\sqrt[5]{(2\text{س}^{3} - 2)^{7}} + \text{جـ}$$

جـ) $\int (2\text{س} - 3\text{س}^{2})\,\text{قا}^{2}(\text{س}^{3} - \text{س}^{2})\,\text{د}\text{س}$

$$\text{ص} = \text{س}^{3} - \text{س}^{2} \Leftarrow \frac{\text{د}\text{ص}}{\text{د}\text{س}} = 3\text{س}^{2} - 2\text{س}$$

$$\frac{\text{د}\text{ص}}{3\text{س}^{2} - 2\text{س}} = \text{د}\text{س}.$$

$$\int \cancel{(2\text{س} - 3\text{س}^{2})}^{\,-1}\,\text{قا}^{2}\text{ص}\,\frac{\text{د}\text{ص}}{\cancel{3\text{س}^{2} - 2\text{س}}}$$

$$\int -\text{قا}^{2}\text{ص}\cdot\text{د}\text{ص} = -\text{ظا}\,\text{ص} + \text{جـ}$$

$$= -\text{ظا}(\text{س}^{3} - \text{س}^{2}) + \text{جـ}$$

د) $\int \frac{3\text{س} - 9}{(\text{س}^{2} - 6\text{س})^{2}}\,\text{د}\text{س}$

$$\text{ص} = \text{س}^{2} - 6\text{س} \Leftarrow \frac{\text{د}\text{ص}}{\text{د}\text{س}} = 2\text{س} - 6 \Leftarrow$$

$$\frac{\text{د}\text{ص}}{2\text{س} - 6} = \text{د}\text{س}.$$

$$\int \frac{3\text{س} - 9}{\text{ص}^{2}} \times \frac{\text{د}\text{ص}}{2\text{س} - 6} =$$

$$\int 3\cancel{(\text{س} - 3)} \times \text{ص}^{-2} \times \frac{\text{د}\text{ص}}{2\cancel{(\text{س} - 3)}} =.$$

$$\frac{3}{2}\,\frac{\text{ص}^{-2+1}}{-2+1} + \text{جـ} = \frac{3}{2}\,\frac{\text{ص}^{-1}}{-1} + \text{جـ}$$

$$\frac{-3}{2\text{ص}} + \text{جـ} = \frac{-3}{2(\text{س}^{2} - 6\text{س})} + \text{جـ}$$

٢) جد قيمة كل من التكاملات الآتية:

أ) $\int \sqrt[3]{(3\text{س}-2)^2}\ \text{دس}$

ب) $\int (\text{س}-1)(2\text{س}^2-4\text{س}+1)^5\ \text{دس}$

جـ) $\int 2\,\text{قا}^2(2-\text{س})\ \text{دس}$

د) $\int 2\text{س}^3\ \text{جا}(\text{س}^4+1)\ \text{دس}$

الحل

أ) $\int \sqrt[3]{(3\text{س}-2)^2}\ \text{دس} = \int (3\text{س}-2)^{\frac{2}{3}}\ \text{دس} = \dfrac{(3\text{س}-2)^{\frac{2}{3}+1}}{(\frac{2}{3}+1)\times 3} + \text{جـ}$

$= \dfrac{(3\text{س}-2)^{\frac{5}{3}}}{\cancel{3}\times\frac{5}{\cancel{3}}} + \text{جـ} = \frac{1}{5}\sqrt[3]{(3\text{س}-2)^5} + \text{جـ}$

ب) $\int (\text{س}-1)(2\text{س}^2-4\text{س}+1)^5\ \text{دس}$

$\text{ص} = 2\text{س}^2-4\text{س}+1 \Leftarrow \dfrac{\text{دص}}{\text{دس}} = 4\text{س}-4$

$\dfrac{\text{دص}}{4\text{س}-4} = \text{دس} \Leftarrow \dfrac{\text{دص}}{4(\text{س}-1)} = \text{دس}$

$\int \cancel{(\text{س}-1)}\ \text{ص}^5 \times \dfrac{\text{دص}}{4\cancel{(\text{س}-1)}} = \frac{1}{4}\int \text{ص}^5\ \text{دص}$

$= \dfrac{\text{ص}^6}{6\times 4} + \text{جـ} = \dfrac{(2\text{س}^2-4\text{س}+1)^6}{24} + \text{جـ}$

جـ) $\int 2\,\text{قا}^2(2-\text{س})\ \text{دس} =$

$\dfrac{2\,\text{ظا}(2-\text{س})}{-1} + \text{جـ}$

$-2\,\text{ظا}(2-\text{س}) + \text{جـ}$

د) $\int 2\text{س}^3\ \text{جا}(\text{س}^4+1)\ \text{دس}$

$\text{ص} = \text{س}^4+1 \Leftarrow \dfrac{\text{دص}}{\text{دس}} = 4\text{س}^3 \Leftarrow$

$\dfrac{\text{دص}}{4\text{س}^3} = \text{دس}$

$\int 2\cancel{\text{س}^3}\ \text{جا ص}\ \dfrac{\text{دص}}{4\cancel{\text{س}^3}} =$

$\int \frac{1}{2}\,\text{جا ص دص} = -\frac{1}{2}\,\text{جتا ص} + \text{جـ}$

$= -\frac{1}{2}\,\text{جتا}(\text{س}^4+1) + \text{جـ}$

٣) احسب قيمة كل من التكاملات الآتية:

أ) $\int_{٠}^{٢} \sqrt{٤\text{س} + ١}\ \text{ءس}$

ب) $\int_{١-}^{١-} ٣\text{س}^{٢}(\text{س}^{٣} - ١)^{٣}\ \text{ءس}$

جـ) $\int_{١}^{٠} ٢\text{س}\sqrt[٣]{\text{س}^{٢} - ١}\ \text{ءس}$

د) $\int_{١}^{٢} \frac{٣ - ٢\text{س}}{(\text{س}^{٢} - ٣\text{س})^{٢}}\ \text{ءس}$

الحل

أ) $\int_{٠}^{٢} \sqrt{٤\text{س}+١}\ \text{ءس} = \int_{٠}^{٢} (٤\text{س}+١)^{\frac{١}{٢}} \cdot \text{ءس}$

$$= \left[\frac{(٤\text{س}+١)^{\frac{١}{٢}+١}}{٤ \times (\frac{١}{٢}+١)} \right]_{٠}^{٢} = \left[\frac{(٤\text{س}+١)^{\frac{٣}{٢}}}{٤ \times \frac{٣}{٢}} \right]_{٠}^{٢}$$

$$\left[\frac{\sqrt{(٤\text{س}+١)^{٣}}}{٦} \right]_{٠}^{٢}$$

$$\frac{١}{٦}\left[\sqrt{(٤\times٢+١)^{٣}} - \sqrt{(٤\times٠+١)^{٣}} \right]$$

$$\frac{١}{٦}\left(\sqrt{٩^{٣}} - \sqrt{١^{٣}} \right)$$

$$\frac{١}{٦}\left(٢٧-١\right)$$

$$\frac{١}{٦}\times ٢٦ = \frac{١٣}{٣}$$

ب) $\int_{-١}^{-١} ٣\text{س}^{٢}\left(\text{س}^{٣}-١\right)^{٣}\,\text{دس} = \text{صفر}$

جـ) $\int_{١}^{٠} ٢\text{س}\,\sqrt[٣]{\text{س}^{٢}-١}\,.\,\text{دس} =$

$$\int_{١}^{٠} ٢\text{س}\left(\text{س}^{٢}-١\right)^{\frac{١}{٣}}\,\text{دس}$$

$$\text{ص} = \text{س}^{٢}-١ \Leftarrow \frac{\text{دص}}{\text{دس}} = ٢\text{س} \Leftarrow \frac{\text{دص}}{٢\text{س}} = \text{دس}$$

$$\int_{١}^{٠} \cancel{٢\text{س}}\,\text{ص}^{\frac{١}{٣}}\,\frac{\text{دص}}{\cancel{٢\text{س}}} = \int_{١}^{٠} \text{ص}^{\frac{١}{٣}}\,\text{دص}$$

$$\left[\frac{\text{ص}^{\frac{١}{٣}+١}}{\frac{١}{٣}+١}\right]_{١}^{٠} = \left[\frac{\text{ص}^{\frac{٤}{٣}}}{\frac{٤}{٣}}\right]_{١}^{٠}$$

$$\left[\frac{٣}{٤}\sqrt[٣]{\left(\text{س}^{٢}-١\right)^{٤}}\right]_{١}^{٠} = \frac{٣}{٤}\left(\sqrt[٣]{\left(٠^{٢}-١\right)^{٤}} - \sqrt[٣]{\left(١-١\right)^{٤}}\right)$$

$\frac{3}{4}\left(\sqrt[3]{1} - \sqrt[3]{0}\right)$

$\frac{3}{4}\left(1 - 0\right)$

$\frac{3}{4} \times 1 = \frac{3}{4}$

د) $\int_{1}^{2} \frac{2س-3}{(س^2-3س)^2}\, دس = \int_{1}^{2} (2س-3)(س^2-3س)^{-2} \cdot دس$

$ص = س^2 - 3س \Leftarrow \frac{دص}{دس} = 2س-3 \Leftarrow دس = \frac{دص}{2س-3}$

$\int_{1}^{2} \cancel{(2س-3)}\, ص^{-2} \times \frac{دص}{\cancel{2س-3}} = \int_{1}^{2} ص^{-2}\, دص =$

$\left[\frac{-ص^{-2+1}}{-2+1}\right]_{1}^{2} = \left[\frac{-ص^{-1}}{-1}\right]_{1}^{2} = \left[\frac{1}{ص}\right]_{1}^{2}$

$\left[\frac{1}{س^2-3س}\right]_{1}^{2} = \frac{1}{2^2-3\times 2} - \frac{1}{1^2-3\times 1} = \frac{1}{-2} - \frac{1}{-2}$

$= \frac{-1}{2} + \frac{1}{2} = صفر$

٤) إذا علمت أن ق(−٨) = ٥، ق(٢٧) = −٦، فجد قيمة التكامل الآتي: $\int_{-2}^{3} 3س^2\, قَ(س^3)\, دس$

الحل

$ص = س^3 \Leftarrow \frac{دص}{دس} = 3س^2 \Leftarrow دس = \frac{دص}{3س^2}$

$\int_{-2}^{3} \cancel{3س^2}\, قَ(ص) \times \frac{دص}{\cancel{3س^2}} = \int_{-2}^{3} قَ(ص) \cdot دص =$

$\left[ق(ص)\right]_{-2}^{3} = \left[ق(س^3)\right]_{-2}^{3}$

$ق(3^3) - ق((-2)^3) = ق(27) - ق(-8) = -6 - 5$

$= -11$

٥) إذا علمت أن $\int_{\text{٥}}^{\text{٢}} \text{ق(س)}\ \text{ءس} = \text{٣}$، فجد قيمة التكامل الآتي: $\int_{\text{١-}}^{\text{٢}} \text{٨س ق(س}^{\text{٢}} + \text{١)}\ \text{ءس}$

الحل

$\text{ص} = \text{س}^{\text{٢}} + \text{١} \Leftarrow \frac{\text{ءص}}{\text{ءس}} = \text{٢س} \Leftarrow \frac{\text{ءص}}{\text{٢س}} = \text{ءس}$

$\int_{\text{١-}}^{\text{٢}} \text{٨}\cancel{\text{س}}\ \text{ق(ص)}\ \frac{\text{ءص}}{\cancel{\text{٢س}}} = \int_{\text{١-}}^{\text{٢}} \text{٤ ق(ص) ءص}$

عندما $\text{س} = \text{١-} \leftarrow \text{ص} = (\text{١-})^{\text{٢}} + \text{١} = \text{٢}$

عندما $\text{س} = \text{٢} \leftarrow \text{ص} = \text{٢}^{\text{٢}} + \text{١} = \text{٥}$

$\int_{\text{٢}}^{\text{٥}} \text{٤ ق(ص) ءص} = \text{٤} \int_{\text{٢}}^{\text{٥}} \text{ق(ص) ءص} = \text{٤} \times \text{٣-} = \text{١٢-}$

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٦) حُلَّ المسألة الواردة في بداية الدرس.

جد قيمة التكامل الآتي:

$$\int_{\text{٠}}^{\text{٤}} \text{٢س} \sqrt{\text{س}^{\text{٢}} + \text{٩}}\ \text{ءس}$$

الحل

$\int_{\text{٠}}^{\text{٤}} \text{٢س}\ (\text{س}^{\text{٢}} + \text{٩})^{\frac{\text{١}}{\text{٢}}} \cdot \text{ءس}$

$\text{ص} = \text{س}^{\text{٢}} + \text{٩} \Leftarrow \frac{\text{ءص}}{\text{ءس}} = \text{٢س} \Leftarrow$

$\frac{\text{ءص}}{\text{٢س}} = \text{ءس}\ .$

$\int_{\text{٠}}^{\text{٤}} \cancel{\text{٢س}} \cdot \text{ص}^{\frac{\text{١}}{\text{٢}}}\ \frac{\text{ءص}}{\cancel{\text{٢س}}} = \int_{\text{٠}}^{\text{٤}} \text{ص}^{\frac{\text{١}}{\text{٢}}} \cdot \text{ءص} = \left[\frac{\text{ص}^{\frac{\text{١}}{\text{٢}}+\text{١}}}{\frac{\text{١}}{\text{٢}}+\text{١}} \right]_{\text{٠}}^{\text{٤}} = \left[\frac{\text{٢}}{\text{٣}} \text{ص}^{\frac{\text{٣}}{\text{٢}}} \right]_{\text{٠}}^{\text{٤}}$

$\frac{\text{٢}}{\text{٣}} \left[\sqrt{(\text{س}^{\text{٢}} + \text{٩})^{\text{٣}}} \right]_{\text{٠}}^{\text{٤}}$

$\frac{\text{٢}}{\text{٣}} \left(\sqrt{(\text{٤}^{\text{٢}} + \text{٩})^{\text{٣}}} - \sqrt{(\text{٠} + \text{٩})^{\text{٣}}} \right)$

$\frac{\text{٢}}{\text{٣}} (\text{٥}^{\text{٣}} - \text{٣}^{\text{٣}}) = \frac{\text{٢}}{\text{٣}} (\text{١٢٥} - \text{٢٧})$

$= \frac{\text{٢}}{\text{٣}} \times \text{٩٨} = \frac{\text{١٩٦}}{\text{٣}}$