إجابات أسئلة الدرس

التكامل بالتعويض

١) اكتب التعويض المناسب لإيجاد قيمة كل تكامل من التكاملات الآتية:

أ) $\int (1 - 2\text{س})(\text{س} - \text{س}^2)^4 \, \text{ءس}$

ب) $\int 6\text{س}^2 \sqrt[5]{(2\text{س}^3 - 2)^2} \, \text{ءس}$

جـ) $\int (2\text{س} - 3\text{س}^2) \, \text{قا}^2(\text{س}^2 - \text{س}^3) \, \text{ءس}$

د) $\int \frac{3\text{س} - 9}{(\text{س}^2 - 6\text{س})^2} \, \text{ءس}$

الحل

أ) $\int (1 - 2\text{س})(\text{س} - \text{س}^2)^4 \, \text{دس}$.

$\text{ص} = \text{س} - \text{س}^2 \Leftarrow \frac{\text{دص}}{\text{دس}} = 1 - 2\text{س} \Leftarrow \frac{\text{دص}}{1 - 2\text{س}} = \text{دس}$

$\int \cancel{(1 - 2\text{س})} \, \text{ص}^4 \frac{\text{دص}}{\cancel{1 - 2\text{س}}} = \int \text{ص}^4 \, \text{دص}$

$= \frac{\text{ص}^5}{5} + \text{جـ} = \frac{(\text{س} - \text{س}^2)^5}{5} + \text{جـ}$

ب) $\int 6\text{س}^2 \sqrt[5]{(2\text{س}^3 - 2)^2} \, \text{دس}$

$\text{ص} = 2\text{س}^3 - 2 \Leftarrow \frac{\text{دص}}{\text{دس}} = 6\text{س}^2 \Leftarrow \frac{\text{دص}}{6\text{س}^2} = \text{دس}$

$\int \cancel{6\text{س}^2} \sqrt[5]{\text{ص}^2} \frac{\text{دص}}{\cancel{6\text{س}^2}} = \int \text{ص}^{\frac{2}{5}} \, \text{دص}$

$$\frac{ص^{\frac{٢}{٥}+١}}{\frac{٢}{٥}+١} + جـ = \frac{ص^{\frac{٧}{٥}}}{\frac{٧}{٥}} + جـ$$

$$= \frac{٥}{٧}\sqrt[٥]{ص^{٧}} + جـ$$

$$= \frac{٥}{٧}\sqrt[٥]{(٢س^{٣} - ٢)^{٧}} + جـ$$

جـ) $\int (٢س - ٣س^{٢})\, قا^{٢}(س^{٣} - س^{٢})\, دس$

$ص = س^{٣} - س^{٢} \Leftarrow \frac{دص}{دس} = ٣س^{٢} - ٢س$

$\frac{دص}{٣س^{٢} - ٢س} = دس$.

$\int \cancel{(٢س - ٣س^{٢})}^{(-١)}\, قا^{٢}ص \, \frac{دص}{\cancel{٣س^{٢} - ٢س}}$

$\int -قا^{٢}ص \cdot دص = -ظا ص + جـ$

$= -ظا(س^{٣} - س^{٢}) + جـ$

د) $\int \frac{٣س - ٩}{(س^{٢} - ٦س)^{٢}}\, دس$

$ص = س^{٢} - ٦س \Leftarrow \frac{دص}{دس} = ٢س - ٦ \Leftarrow$

$\frac{دص}{٢س - ٦} = دس$.

$\int \frac{٣س - ٩}{ص^{٢}} \times \frac{دص}{٢س - ٦} =$

$\int ٣\cancel{(س - ٣)} \times ص^{-٢} \times \frac{دص}{٢\cancel{(س - ٣)}} =$.

$\frac{٣}{٢}\, \frac{ص^{-٢+١}}{-٢+١} + جـ = \frac{٣}{٢}\, \frac{ص^{-١}}{-١} + جـ$

$\frac{-٣}{٢ص} + جـ = \frac{-٣}{٢(س^{٢} - ٦س)} + جـ$

٢) جد قيمة كل من التكاملات الآتية:

أ) $\int \sqrt[3]{(3س - 2)^2}\, ءس$ ب) $\int (س - 1)(2س^2 - 4س + 1)^5\, ءس$

جـ) $\int 2\,قا^2(2 - س)\, ءس$ د) $\int 2س^3\, جا(س^4 + 1)\, ءس$

الحل

أ) $\int \sqrt[3]{(3س-2)^2}\, ءس = \int (3س-2)^{\frac{2}{3}}\, ءس = \dfrac{(3س-2)^{\frac{2}{3}+1}}{(\frac{2}{3}+1)\times 3} + جـ$

$= \dfrac{(3س-2)^{\frac{5}{3}}}{\cancel{3} \times \frac{5}{\cancel{3}}} + جـ = \frac{1}{5}\sqrt[3]{(3س-2)^5} + جـ$

ب) $\int (س-1)(2س^2 - 4س + 1)^5\, ءس$

$ص = 2س^2 - 4س + 1 \Leftarrow \dfrac{ءص}{ءس} = 4س - 4$

$\dfrac{ءص}{4س-4} = ءس \Leftarrow \dfrac{ءص}{4(س-1)} = ءس$

$\int \cancel{(س-1)}\, ص^5 \times \dfrac{ءص}{4\cancel{(س-1)}} = \frac{1}{4}\int ص^5\, ءص$

$= \dfrac{ص^6}{6\times 4} + جـ = \dfrac{(2س^2 - 4س + 1)^6}{24} + جـ$

جـ) $\int 2\,قا^2(2-س)\, ءس =$

$2\,\dfrac{ظا(2-س)}{-1} + جـ$

$-2\,ظا(2-س) + جـ$

د) $\int 2س^3\, جا(س^4+1)\, ءس$

$ص = س^4 + 1 \Leftarrow \dfrac{ءص}{ءس} = 4س^3 \Leftarrow$

$\dfrac{ءص}{4س^3} = ءس$

$\int 2\cancel{س^3}\, جا ص\, \dfrac{ءص}{4\cancel{س^3}} =$

$\int \frac{1}{2}\, جا ص\, ءص = -\frac{1}{2}\, جتا ص + جـ$

$= -\frac{1}{2}\, جتا(س^4+1) + جـ$

٣) احسب قيمة كل من التكاملات الآتية:

أ) $\int_{0}^{2} \sqrt{4\text{س} + 1}\ \text{دس}$

ب) $\int_{-1}^{-1} 3\text{س}^2(\text{س}^3 - 1)^3\ \text{دس}$

جـ) $\int_{1}^{} 2\text{س} \sqrt[3]{\text{س}^2 - 1}\ \text{دس}$

د) $\int_{1}^{2} \frac{3 - 2\text{س}}{(\text{س}^2 - 3\text{س})^2}\ \text{دس}$

الحل

أ) $\int_{0}^{2} \sqrt{4\text{س}+1}\ \text{دس} = \int_{0}^{2} (4\text{س}+1)^{\frac{1}{2}} \cdot \text{دس}$

$$= \left[\frac{(4\text{س}+1)^{\frac{1}{2}+1}}{4 \times (\frac{1}{2}+1)} \right]_{0}^{2} = \left[\frac{(4\text{س}+1)^{\frac{3}{2}}}{4 \times \frac{3}{2}} \right]_{0}^{2}$$

$$\left[\frac{\sqrt{(4\text{س}+1)^3}}{6} \right]_{0}^{2}$$

$$\frac{1}{6}\left[\sqrt{(4\times 2+1)^3} - \sqrt{(4 \times 0 + 1)^3} \right]$$

$$\frac{1}{6}\left(\sqrt{9}^{\,3} - \sqrt{1}^{\,3} \right)$$

$$\frac{1}{6}(27-1)$$

$$\frac{1}{6}\times 26 = \frac{13}{3}$$

ب) $\int_{-1}^{1} 3س^{2}(س^{3}-1)^{3}\, دس = صفر$

جـ) $\int_{1}^{0} 2س\sqrt[3]{س^{2}-1}\, دس =$

$$\int_{1}^{0} 2س\,(س^{2}-1)^{\frac{1}{3}}\, دس$$

$$ص = س^{2}-1 \Leftarrow \frac{دص}{دس} = 2س \Leftarrow دس = \frac{دص}{2س}$$

$$\int_{1}^{0} \cancel{2س}\, ص^{\frac{1}{3}} \frac{دص}{\cancel{2س}} = \int_{1}^{0} ص^{\frac{1}{3}}\, دص$$

$$\left[\frac{ص^{\frac{1}{3}+1}}{\frac{1}{3}+1}\right]_{1}^{0} = \left[\frac{ص^{\frac{4}{3}}}{\frac{4}{3}}\right]_{1}^{0}$$

$$\frac{3}{4}\left[\sqrt[3]{(س^{2}-1)^{4}}\right]_{1}^{0} = \frac{3}{4}\left(\sqrt[3]{(0^{2}-1)^{4}} - \sqrt[3]{(1-1)^{4}}\right)$$

$$\frac{٣}{٤}\left(\sqrt[٣]{١} - \sqrt[٣]{٠}\right)$$

$$\frac{٣}{٤}\left(١ - ٠\right)$$

$$\frac{٣}{٤} \times ١ = \frac{٣}{٤}$$

د) $\int_{١}^{٢} \frac{٢س - ٣}{(س^{٢} - ٣س)^{٢}} \, دس = \int_{١}^{٢} (٢س - ٣)(س^{٢} - ٣س)^{-٢} \, . \, دس$

$ص = س^{٢} - ٣س \Leftarrow \frac{دص}{دس} = ٢س - ٣ \Leftarrow دس = \frac{دص}{٢س - ٣}$

$\int_{١}^{٢} \cancel{(٢س - ٣)} \, ص^{-٢} \times \frac{دص}{\cancel{٢س - ٣}} = \int_{١}^{٢} ص^{-٢} \, دص =$

$\left[\frac{-ص^{-٢+١}}{-٢+١}\right]_{١}^{٢} = \left[\frac{-ص^{-١}}{-١}\right]_{١}^{٢} = \left[\frac{١}{ص}\right]_{١}^{٢}$

$\left[\frac{١}{س^{٢} - ٣س}\right]_{١}^{٢} = \frac{١}{٢^{٢} - ٣ \times ٢} - \frac{١}{١^{٢} - ٣ \times ١} = \frac{١}{-٢} - \frac{١}{-٢}$

$= \frac{-١}{٢} + \frac{١}{٢} = صفر$

٤) إذا علمت أن ق(−٨) = ٥، ق(٢٧) = −٦، فجد قيمة التكامل الآتي: $\int_{-٢}^{٣} ٣س^{٢} \, قَ(س^{٣}) \, دس$

الحل

$ص = س^{٣} \Leftarrow \frac{دص}{دس} = ٣س^{٢} \Leftarrow دس = \frac{دص}{٣س^{٢}}$

$\int_{-٢}^{٣} \cancel{٣س^{٢}} \, قَ(ص) \, \frac{دص}{\cancel{٣س^{٢}}} = \int_{-٢}^{٣} قَ(ص) \, . \, دص =$

$\left[ق(ص)\right]_{-٢}^{٣} = \left[ق(س^{٣})\right]_{-٢}^{٣}$

$ق(٣^{٣}) - ق((-٢)^{٣}) = ق(٢٧) - ق(-٨) = -٦ - ٥$

$= -١١$

٥) إذا علمت أن $\int_{\text{٥}}^{\text{٢}} \text{ق(س)}\ \text{دس} = \text{٣}$، فجد قيمة التكامل الآتي: $\int_{\text{١-}}^{\text{٢}} \text{٨س ق(س}^{\text{٢}} + \text{١)}\ \text{دس}$

الحل

$$\text{ص} = \text{س}^{\text{٢}} + \text{١} \Leftarrow \frac{\text{دص}}{\text{دس}} = \text{٢س} \Leftarrow \frac{\text{دص}}{\text{٢س}} = \text{دس}$$

$$\int_{\text{١-}}^{\text{٢}} \text{٨}\cancel{\text{س}}\ \text{ق(ص)} \frac{\text{دص}}{\cancel{\text{٢س}}} = \text{٤} \int_{\text{١-}}^{\text{٢}} \text{ق(ص)}\ \text{دص}$$

$$\text{عندما س} = \text{١-} \leftarrow \text{ص} = (\text{١-})^{\text{٢}} + \text{١} = \text{٢}$$

$$\text{عندما س} = \text{٢} \leftarrow \text{ص} = \text{٢}^{\text{٢}} + \text{١} = \text{٥}$$

$$\text{٤} \int_{\text{٢}}^{\text{٥}} \text{ق(ص)}\ \text{دص} = \text{٤} \int_{\text{٢}}^{\text{٥}} \text{ق(ص)}\ \text{دص} = \text{٤} \times \text{٣-} = \text{١٢-}$$

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٦) حُلَّ المسألة الواردة في بداية الدرس.

جد قيمة التكامل الآتي:

$$\int_{\text{٠}}^{\text{٤}} \text{٢س} \sqrt{\text{س}^{\text{٢}} + \text{٩}}\ \text{دس}$$

الحل

$$\int_{\text{٠}}^{\text{٤}} \text{٢س} \ (\text{س}^{\text{٢}} + \text{٩})^{\frac{\text{١}}{\text{٢}}} \cdot \text{دس}$$

$$\text{ص} = \text{س}^{\text{٢}} + \text{٩} \Leftarrow \frac{\text{دص}}{\text{دس}} = \text{٢س} \Leftarrow$$

$$\frac{\text{دص}}{\text{٢س}} = \text{دس} \ .$$

$$\int_{\text{٠}}^{\text{٤}} \cancel{\text{٢س}} \cdot \text{ص}^{\frac{\text{١}}{\text{٢}}} \frac{\text{دص}}{\cancel{\text{٢س}}} = \int_{\text{٠}}^{\text{٤}} \text{ص}^{\frac{\text{١}}{\text{٢}}} \cdot \text{دص} = \left[\frac{\text{ص}^{\frac{\text{١}}{\text{٢}}+\text{١}}}{\frac{\text{١}}{\text{٢}}+\text{١}} \right]_{\text{٠}}^{\text{٤}} = \left[\frac{\text{٢}}{\text{٣}} \text{ص}^{\frac{\text{٣}}{\text{٢}}} \right]_{\text{٠}}^{\text{٤}}$$

$$\left[\frac{\text{٢}}{\text{٣}} \sqrt{(\text{س}^{\text{٢}} + \text{٩})^{\text{٣}}} \right]_{\text{٠}}^{\text{٤}}$$

$$\frac{\text{٢}}{\text{٣}} \left(\sqrt{(\text{٤}^{\text{٢}} + \text{٩})^{\text{٣}}} - \sqrt{(\text{٠} + \text{٩})^{\text{٣}}} \right)$$

$$\frac{\text{٢}}{\text{٣}} \left(\text{٥}^{\text{٣}} - \text{٣}^{\text{٣}} \right) = \frac{\text{٢}}{\text{٣}} (\text{١٢٥} - \text{٢٧})$$

$$= \frac{\text{٢}}{\text{٣}} \times \text{٩٨} = \frac{\text{١٩٦}}{\text{٣}}$$