

أدرب وأحل المسائل

العمليات على الأعداد المركبة

أجد ناتج كلٍّ ممّا يأتي، ثم أكتبه بالصورة القياسية:

$$1) (7 + 2i) + (3 - 11i)$$

$$(7+2i)+(3-11i)=10-9i$$

$$(2) (5 - 9i) - (-4 + 7i)$$

$$(5-9i)-(-4+7i)=9-16i$$

$$(3) (4 - 3i) (1 + 3i)$$

$$(4-3i)(1+3i)=4+12i-3i+9=13+9i$$

$$(4) (4 - 6i) (1 - 2i) (2 - 3i)$$

$$(4-6i)(1-2i)(2-3i)=(4-6i)(2-3i-4i-6)=(4-6i)(-4-7i)=-16-28i+24i-42=-58-4i$$

$$(5) (x^2 + y^2)^2 = 50(x^2 - y^2)$$

$$(9-2i)^2=81-36i-4=77-36i$$

$$(6) 48+19i^5-4i$$

$$48+19i^5-4i=48+19i^5-4i \times 5+4i^5+4i=240+192i+95i-7625+16=164+287i41=4+7i$$

أجد ناتج كلٍّ ممّا يأتي بالصورة المثلثية:

$$(7) 6(\cos \pi + i \sin \pi) \times 2(\cos (-\pi/4) + i \sin (-\pi/4))$$

$$6(\cos \pi + i \sin \pi) \times 2(\cos (-\pi/4) + i \sin (-\pi/4)) = 12(\cos (\pi - \pi/4) + i \sin (\pi - \pi/4)) = 12(\cos 3\pi/4 + i \sin 3\pi/4)$$

$$(8) (\cos 3\pi 10 + i \sin 3\pi 10) \div (\cos 2\pi 5 + i \sin 2\pi 5)$$

$$(\cos (3\pi 10) + i \sin (3\pi 10)) \div (\cos 2\pi 5 + i \sin 2\pi 5) = \cos (3\pi 10 - 2\pi 5) + i \sin (3\pi 10 - 2\pi 5) = \cos (-\pi 10) + i \sin (-\pi 10)$$

$$(9) 12(\cos \pi 4 + i \sin \pi 4) \div 4(\cos \pi 3 + i \sin \pi 3)$$

$$12(\cos (\pi 4) + i \sin (\pi 4)) \div 4(\cos \pi 3 + i \sin \pi 3) = 124(\cos (\pi 4 - \pi 3) + i \sin (\pi 4 - \pi 3)) = 3(\cos (-\pi 12) + i \sin (-\pi 12))$$

$$(10) 11(\cos (-\pi 6) + i \sin (-\pi 6)) \times 2(\cos 3\pi 2 + i \sin 3\pi 2)$$

$$11(\cos(-\pi 6) + i \sin(-\pi 6)) \times 2(\cos(3\pi 2) + i \sin(3\pi 2)) = 22(\cos(-\pi 6 + 3\pi 2) + i \sin(-\pi 6 + 3\pi 2)) = 22(\cos (4\pi 3) + i \sin (4\pi 3)) = 22(\cos (4\pi 3 - 2\pi) + i \sin (4\pi 3 - 2\pi)) = 22(\cos (-2\pi 3) + i \sin (-2\pi 3))$$

أجد القيم الحقيقية للثابتين a و b في كل مما يأتي:

$$(11) (a + 6i) + (7 - ib) = -2 + 5i$$

$$(a + 6i) + (7 - bi) = -2 + 5i \rightarrow a + 7 + (6 - b)i = -2 + 5i \rightarrow a + 7 = -2, 6 - b = 5 \rightarrow a = -9, b = 1$$

$$(12) (11 - ia) - (b - 9i) = 7 - 6i$$

$$(11 - ia) - (b - 9i) = 7 - 6i \rightarrow 11 - b + (9 - a)i = 7 - 6i \rightarrow 11 - b = 7, 9 - a = -6$$

$$(13) (a + ib)(2 - i) = 5 + 5i$$

$$(a + ib)(2 - i) = 5 + 5i \rightarrow 2a + b + (2b - a)i = 5 + 5i \rightarrow 2a + b = 5, 2b - a = 5 \rightarrow b = 3, a = 1$$

طريقة ثانية للحل:

$$a + ib = 5 + 5i \rightarrow 2 - i = 5 + 5i \rightarrow 2 - i \times 2 + i \times 2 + i = 10 + 5i + 10i - 5 + 1 = 5 + 15i \rightarrow a = 1 + 3i \rightarrow a = 1, b = 3$$

$$(14) a - 6i = 1 - 2i = b + 4i$$

$$a-6i1-2i=b+4i \rightarrow a-6i1-2i \times 1+2i1+2i=b+4i \rightarrow a+2ai-6i+121+4=b+4i \rightarrow a+125+2a-65i=b+4i \rightarrow a+125=b, 2a-65=4 \rightarrow a=13$$

a بتعويض قيمة في المعادلة الأولى ينتج أن: $b = 5$

طريقة ثانية للحل:

$$a-6i=(b+4i)(1-2i) \rightarrow a-6i=b+8+(-2b+4)i \rightarrow a=b+8, -6=-2b+4 \rightarrow b=5, a=13$$

(15) أضرب العدد المركب $8(\cos(\pi/4) - i\sin(\pi/4))$ في مرافقه.

$$z=8(\cos(\pi/4)-i\sin(\pi/4))=8(\cos(-\pi/4)+i\sin(-\pi/4)) \rightarrow z^{-}=8(\cos(\pi/4)+i\sin(\pi/4)) \rightarrow zz^{-}=8(\cos(\pi/4)-i\sin(\pi/4)) \times 8(\cos(\pi/4)+i\sin(\pi/4))=64(\cos^2(\pi/4)+\sin^2(\pi/4))=64$$

الحل الثاني: نكتب كلاً من العددين بالصورة المثلثية أولاً ثم نطبق القاعدة:

$$z=8(\cos(\pi/4)-i\sin(\pi/4))=8(\cos(-\pi/4)+i\sin(-\pi/4)) \rightarrow z^{-}=8(\cos(\pi/4)+i\sin(\pi/4)) \rightarrow zz^{-}=64(\cos(-\pi/4+\pi/4)+i\sin(-\pi/4+\pi/4))=64$$

الحل الثالث: نكتب كلاً من العددين بالصورة القياسية أولاً ثم إجراء عملية الضرب:

$$z=8(\cos(\pi/4)-i\sin(\pi/4))=8(12-12i)=42-42i \rightarrow z^{-}=42+42i \rightarrow zz^{-}=(42-42i)(42+42i)=32+32=64$$

إذا كان: $z_1=12-2i, z_2=5-i, z_3=2-2i$ ، فأجد المقياس والسعة الرئيسة لكل مما يأتي:

(16) z_2z_1

$$z_1=23-2i, z_2=5-i, z_3=2-2i |z_1|=12+4=4 |z_2|=5+15=25 |z_3|=4+4=22 \text{Arg}(z_1)=-\tan^{-1}(2/3)=-\tan^{-1}(1/3)=-\pi/6 \text{Arg}(z_2)=-\tan^{-1}(1/5)=-\tan^{-1}(1/3)=-\pi/3 \text{Arg}(z_3)=-\tan^{-1}(2/2)=-\tan^{-1}(1)=-\pi/4 |z_2z_1|=|z_2||z_1|=25 \times 4=100 \text{Arg}(z_2z_1)=\text{Arg}(z_2)+\text{Arg}(z_1)=-\pi/3+(-\pi/6)=-\pi/2$$

(17) $1z_3$

$$|1z_3|=|1||z_3|=22 \text{Arg}(1z_3)=\text{Arg}(1)+\text{Arg}(z_3)=0+(-\pi/4)=-\pi/4$$

(18) $z^3 z^2$

$$z^2 = 5 + i15 \rightarrow |z^2| = |z|^2 = 25, \text{Arg}(z^2) = \pi/3 \quad |z^3 z^2| = |z^3| |z^2| \\ = 2 \cdot 25 = 50 \quad \text{Arg}(z^3 z^2) = \text{Arg}(z^3) - \text{Arg}(z^2) = -\pi/4 - \pi/3 = -7\pi/12$$