

$$\int x dx \quad (7 \sec^4 f)$$

$$x \Rightarrow du dx = \sec x) dx u = \tan x (1 + \tan^2 x dx = \int \sec^2 x \times \sec^2 x dx = \int \sec^2 \sec^4 f$$

$$x = \int (1 + u^2) du = u + \frac{1}{3} u^3 + C = \tan x + \frac{1}{3} \tan^3 x + C = \tan$$

$$\int x dx \quad (8 x \cos^2 \tan f)$$

$$x \int \tan x \Rightarrow dx = du \sec^2 x \Rightarrow du dx = \sec^2 x dx u = \tan x \sec^2 x dx = \int \tan x \cos^2 \tan f$$

$$x + C x = \int u du = \frac{1}{2} u^2 + C = \frac{1}{2} \tan^2 x \times du \sec^2 x dx = \int u \sec^2 x \cos^2 n$$

$$\int x dx \quad (9 (\ln \sin f)$$

$$u du = -\cos u x \times x du = \int \sin x) x dx = \int \sin (\ln x \Rightarrow du dx = \frac{1}{x} \Rightarrow dx = x du \int \sin u = \ln$$

$$x) + C (\ln u + C = -\cos$$

$$\int x dx \quad (10 x^1 + \sin^2 x \cos \sin f)$$

$$x) + C (1 + \sin^2 x dx = \frac{1}{2} \ln x^1 + \sin^2 x \cos x dx = \frac{1}{2} \int 2 \sin x^1 + \sin^2 x \cos \sin f$$

$$\int (2e^x - 2e^{-x})(e^x + e^{-x})^2 dx \quad (11 f)$$

$$u = e^x + e^{-x} \Rightarrow du dx = e^x - e^{-x} \Rightarrow dx = du e^x - e^{-x} \int 2e^x - 2e^{-x} (e^x + e^{-x})^2 d$$

$$x = \int 2(e^x - e^{-x}) u^2 \times du e^x - e^{-x} = \int 2u^2 - 2 du = -\frac{2}{3} u^3 + C = -\frac{2}{3} (e^x + e^{-x})^3 + C$$

$$\int x(x+1)^{x+1} dx \quad (12 - f)$$

$$u = x+1 \Rightarrow dx = du, x = u-1 \int -x(x+1)^{x+1} dx = \int 1-u u^u du = \int 1-u u^3 du = \int$$

$$(u^3 - 3u^2 - u - 1) du = \frac{1}{4} u^4 - \frac{3}{3} u^3 - \frac{1}{2} u^2 - u + C = \frac{1}{4} (x+1)^4 - (x+1)^3 - \frac{1}{2} (x+1)^2 - (x+1) + C =$$

$$-\frac{1}{4} x^4 + \frac{1}{2} x^3 - \frac{1}{4} x^2 + \frac{1}{2} x + C$$

$$\int x(x+10)^3 dx \quad (13 f)$$

$$u = x+10 \Rightarrow dx = du, x = u-10 \int x(x+10)^3 dx = \int (u-10) u^3 du = \int (u^4 - 10u^3$$

$$) du = \frac{1}{5} u^5 - 10 \frac{1}{4} u^4 + C = \frac{1}{5} (x+10)^5 - \frac{5}{2} (x+10)^4 + C = \frac{1}{5} (x+10)^5 - \frac{5}{2} (x+10)^4 + C$$

$$\int x^2 dx \quad (14 x^2 \tan^7 \sec^2) f$$

$$x^2 dx = \int \sec^2 x \tan^7 x^2 \int \sec^2 x^2 \Rightarrow dx = 2 du \sec^2 x^2 \Rightarrow du dx = 12 \sec^2 u = \tan x^2 + C x^2 = 2 \int u^7 du = 14 u^8 + C = 14 \tan^8 x^2 u^7 \times 2 du \sec^2$$

$$(x dx (15 x \sec x + e \sin \sec^3 \int$$

$$x x e \sin x dx + \int \cos x) dx = \int \sec^2 x e \sin x + \cos x dx = \int (\sec^2 x \sec x + e \sin \sec^3 \int x dx + x dx = \int \sec^2 x \sec x + e \sin x \int \sec^3 x \Rightarrow dx = du \cos x \Rightarrow du dx = \cos dx u = \sin x + C x + e \sin x + e u + C = \tan x + \int e u du = \tan x = \tan x e u \times du \cos \int \cos$$

$$(x dx (16 x^3) \cos^3 \sin + 1) \int$$

$$x dx = \int (1 + u^3) \cos^3 x^3) \cos^3 x \int (1 + \sin x \Rightarrow dx = du \cos x \Rightarrow du dx = \cos u = \sin x) du = \int (1 + u^3) (1 - u^2) du = \int (1 + u^3) (1 - \sin^2 x) = \int (1 + u^3) \cos^2 x du \cos) du = \int (1 + u^3) (1 - u^2) du = \int (1 - u^2 + u^3 - u^7) du = u - \frac{1}{3} u^3 + \frac{3}{4} u^4 - \frac{1}{8} u^8 + C x - \frac{1}{3} \sin^3 x + \frac{3}{4} \sin^4 x - \frac{1}{8} \sin^8 x + C = \sin$$

$$(x dx (17 x \sec^5 \sin \int$$

$$x \int \sin x \Rightarrow dx = du - \sin x \Rightarrow du dx = -\sin x dx u = \cos x \cos - 5 x dx = \int \sin x \sec^5 \sin \int x + x = - \int u - 5 du = 14 u - 4 + C = 14 \cos - 4 x u - 5 \times du - \sin x dx = \int \sin x \sec^5 n x + C C = 14 \sec^4$$

$$(x dx (18 x \cos^3 x + \tan \sin \int$$

$$x + s x (\sec x \sec x) dx = \int \tan x \sec^3 x + \tan x \sec^2 x dx = \int (\tan x \cos^3 x + \tan \sin \int x dx \cos^3 x + \tan x \int \sin x \sec x \Rightarrow dx = du \tan x \sec x \Rightarrow du dx = \tan x) dx u = \sec^2 x = \int (u + u^2) du = 12 u^2 + 13 u^3 + C = 12 \sec x \sec x (u + u^2) du \tan x \sec x = \int \tan x + C x + 13 \sec^3 2$$

أجد قيمة كلا من التكمالات الآتية:

$$(2 x dx (19 x^{1 - \cos 20\pi/4} \sin \int$$

$$|2 x^2 x| = |\sin^2 x = \sin^2 \cos^2 - 1$$

لكن الزاوية $2x$ تكون ضمن الربع الأول عندما $0 < 2x < \pi/4$

لذا فإن $2x > 0 \sin$ ويكون $2x^2 x| = \sin \sin$

$$x \Rightarrow x dx u = \sin x \cos 2x dx = \int_0^{\pi/4} 2 \sin 2x \sin 2x dx = \int_0^{\pi/4} 2 \sin^2 x dx = \int_0^{\pi/4} 2(1 - \cos 2x) dx = 2x - \sin 2x \Big|_0^{\pi/4} = \left(\frac{\pi}{2} - 1\right) - (0 - 0) = \frac{\pi}{2} - 1$$

$$(x^2 dx) \int_0^{200\pi/2} x \sin x dx$$

$$x^2 dx = \int_0^{\pi/2} u = x^2 \Rightarrow du dx = 2x \Rightarrow dx = \frac{du}{2x} \quad x = \frac{\pi}{2} \Rightarrow u = \frac{\pi^2}{4} \quad x = 0 \Rightarrow u = 0$$

$$\int_0^{\pi/2} x^2 \sin x dx = \int_0^{\pi^2/4} \frac{u}{2} \sin \sqrt{u} \frac{du}{2\sqrt{u}} = \frac{1}{4} \int_0^{\pi^2/4} \sqrt{u} \sin \sqrt{u} du$$

$$(01x^3 + 1 + x^2 dx) \int_0^1 (21f)$$

$$u = 1 + x^2 \Rightarrow du dx = 2x \Rightarrow dx = \frac{du}{2x} \quad x^2 = u - 1 \quad x = 0 \Rightarrow u = 1 \quad x = 1 \Rightarrow u = 2$$

$$\int_0^1 (01x^3 + 1 + x^2) dx = \int_1^2 \left(\frac{u-1}{2} + 1 + \frac{u-1}{2}\right) \frac{du}{2\sqrt{u-1}} = \frac{1}{4} \int_1^2 (u-1+2+u-1) du = \frac{1}{4} \int_1^2 (2u) du = \frac{1}{2} \int_1^2 u du = \frac{1}{2} \left(\frac{u^2}{2}\right) \Big|_1^2 = \frac{1}{4} (4 - 1) = \frac{3}{4}$$

$$(x dx) \int_0^{22} x \tan 50\pi/3 \sec^2 x dx$$

$$x \tan x = 0 \Rightarrow u = 0 \quad x = \pi/3 \Rightarrow u = 3 \quad \int_0^{\pi/3} x \tan x \sec^2 x dx = \int_0^3 u \tan \sqrt{u} \sec^2 \sqrt{u} \frac{du}{2\sqrt{u}} = \frac{1}{2} \int_0^3 \sqrt{u} \tan \sqrt{u} \sec^2 \sqrt{u} du$$

$$(x-1)e^{(x-1)^2} dx \int_0^2 (23)02f$$

$$u = (x-1)^2 \Rightarrow du dx = 2(x-1) \Rightarrow dx = \frac{du}{2(x-1)} \quad x = 0 \Rightarrow u = 1 \quad x = 2 \Rightarrow u = 1$$

$$\int_0^2 (x-1)e^{(x-1)^2} dx = \int_1^1 \frac{1}{2} e^u du = 0$$

$$(x dx) \int_0^2 (24+142f)$$

$$u = 2 + x \Rightarrow du dx = 1 \Rightarrow dx = du \quad x = 0 \Rightarrow u = 2 \quad x = 2 \Rightarrow u = 4$$

$$\int_0^2 (24 + 14x^2) dx = \int_2^4 (24 + 14(u-2)^2) du = \int_2^4 (24 + 14(u^2 - 4u + 4)) du = \int_2^4 (14u^2 - 42u + 40) du = \left(\frac{14}{3}u^3 - 21u^2 + 40u\right) \Big|_2^4 = \left(\frac{14}{3}(64) - 21(16) + 160\right) - \left(\frac{14}{3}(8) - 21(4) + 80\right) = \left(\frac{896}{3} - 336 + 160\right) - \left(\frac{112}{3} - 84 + 80\right) = \frac{784}{3} - 252 + 80 = \frac{784 - 756 + 240}{3} = \frac{268}{3}$$

$$(0110x(1+x^3)^2 dx) \int_0^1 (25f)$$

$$u = 1 + x^3 \Rightarrow du dx = 3x^2 \Rightarrow dx = \frac{du}{3x^2} \quad x = 0 \Rightarrow u = 1 \quad x = 1 \Rightarrow u = 2$$

$$\int_0^1 10x(1+x^3)^2 dx = \int_1^2 \frac{10}{3} \sqrt[3]{u} (u)^2 \frac{du}{3\sqrt[3]{u}} = \frac{10}{9} \int_1^2 u^{5/3} du = \frac{10}{9} \left(\frac{3}{8}u^{8/3}\right) \Big|_1^2 = \frac{10}{9} \left(\frac{3}{8}(2^{8/3}) - \frac{3}{8}(1^{8/3})\right) = \frac{10}{9} \left(\frac{3}{8}(16) - \frac{3}{8}\right) = \frac{10}{9} \left(\frac{48}{8} - \frac{3}{8}\right) = \frac{10}{9} \left(\frac{45}{8}\right) = \frac{50}{6}$$

$$(x dx) \int_0^{26} x \sin 0\pi/6 \cos x dx$$

$$x=0 \Rightarrow u=1 \quad x=\pi/6 \Rightarrow u=3/2 \quad \int_0^{\pi/6} 2 \cos x \Rightarrow dx = du - \sin x \Rightarrow du dx = -\sin u = \cos 2$$

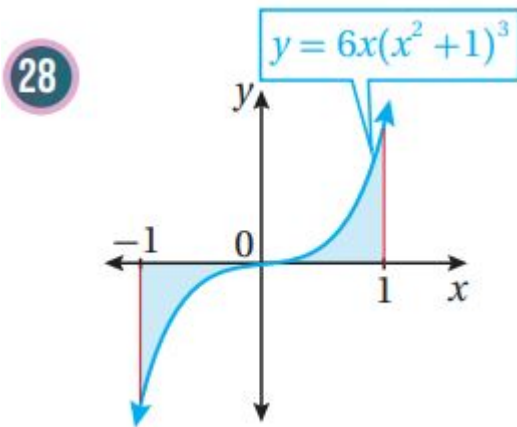
$$2(2322|132 = -1 \ln x = -\int 1322 u du = -2u \ln x du - \sin x dx = \int 1322 u \sin x \sin -2) \approx 0.256$$

$$\int x dx (27x \cot 5\pi/4 \pi/2 \csc 2 \int$$

$$x=\pi/2 \Rightarrow u=0 \quad x=\pi/4 \Rightarrow u=1 \quad \int_{\pi/4}^{\pi/2} 2x \Rightarrow dx = du - \csc 2x \Rightarrow du dx = -\csc 2u = \cot x$$

$$x = \int 10 - u^5 du = -16u^6 |_{10} = 16xu^5 du - \csc 2x dx = \int 10 \csc 2x \cot 5 \csc 2$$

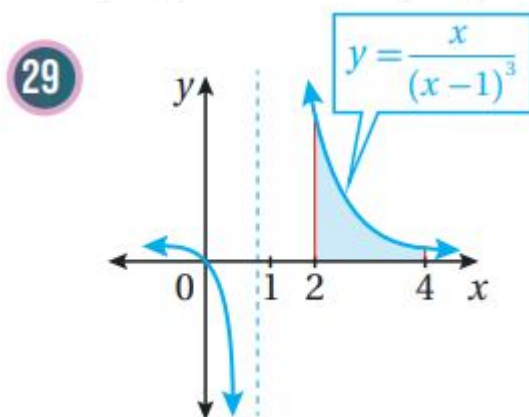
أجد مساحة المنطقة المظللة في كل من التمثيلات البيانية الآتية:



$$A = -\int_{-1}^0 6x(x^2+1)^3 dx + \int_0^1 6x(x^2+1)^3 dx \quad u = x^2+1 \Rightarrow du dx = 2x \Rightarrow dx = du / 2x$$

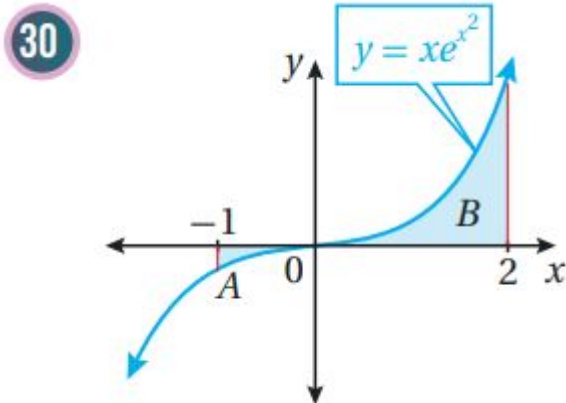
$$u^2 x x = -1 \Rightarrow u = 2x = 0 \Rightarrow u = 1 \quad x = 1 \Rightarrow u = 2 \quad A = -\int_2^1 16xu^3 du + \int_1^2 126xu^3 du$$

$$x = \int 123u^3 du + \int 123u^3 du = \int 126u^3 du = 64u^4 |_{12} = 452$$



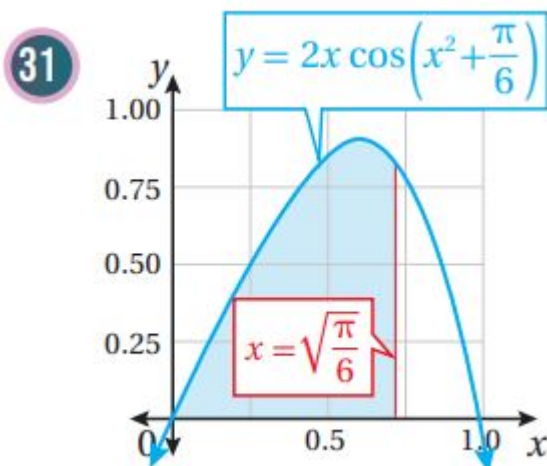
$$A = \int_2^4 \frac{x}{(x-1)^3} dx \quad u = x-1 \Rightarrow dx = du, x = u+1 \quad x=2 \Rightarrow u=1 \quad x=4 \Rightarrow u=3 \quad A = \int_1^3 \frac{24x}{(x-1)^3} dx$$

$$(x-1)^3 dx = \int 13u + 1u^3 du = \int 13(u-2+u-3) du = (-u-1-12u-2) |_{13} = -13 - 12(19) + 1 + 12 = 109$$



$$u = x^2 \Rightarrow \frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x} \Rightarrow \frac{dx}{x} = \frac{du}{2u} \Rightarrow \int \frac{dx}{x} = \frac{1}{2} \int \frac{du}{u} \Rightarrow \ln|x| = \frac{1}{2} \ln|u| \Rightarrow \ln|x| = \frac{1}{2} \ln|x^2| \Rightarrow \ln|x| = \ln|x|$$

$$A = \int_{-1}^0 x e^{x^2} dx + \int_0^2 x e^{x^2} dx = \int_{-1}^0 \frac{1}{2} e^u du + \int_0^2 \frac{1}{2} e^u du = \frac{1}{2} [e^u]_{-1}^0 + \frac{1}{2} [e^u]_0^2 = \frac{1}{2} (e^0 - e^{-1}) + \frac{1}{2} (e^2 - e^0) = \frac{1}{2} (1 - e^{-1} + e^2 - 1) = \frac{1}{2} (e^2 - e^{-1}) \approx 27.658$$



$$u = x^2 + \frac{\pi}{6} \Rightarrow \frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x} \Rightarrow \frac{dx}{x} = \frac{du}{2u} \Rightarrow \int \frac{dx}{x} = \frac{1}{2} \int \frac{du}{u} \Rightarrow \ln|x| = \frac{1}{2} \ln|u| \Rightarrow \ln|x| = \frac{1}{2} \ln|x^2 + \frac{\pi}{6}|$$

$$A = \int_0^{\sqrt{\pi/6}} 2x \cos(x^2 + \frac{\pi}{6}) dx + \int_{\sqrt{\pi/6}}^1 2x \cos(x^2 + \frac{\pi}{6}) dx = \int_{\pi/6}^{\pi/3} \cos u du + \int_{\pi/3}^{\pi/2} \cos u du = [\sin u]_{\pi/6}^{\pi/3} + [\sin u]_{\pi/3}^{\pi/2} = (\sin \frac{\pi}{3} - \sin \frac{\pi}{6}) + (\sin \frac{\pi}{2} - \sin \frac{\pi}{3}) = (\frac{\sqrt{3}}{2} - \frac{1}{2}) + (1 - \frac{\sqrt{3}}{2}) = 3 - \sqrt{3} \approx 0.366$$

في كل مما يأتي المشتقة الأولى للاقتران $(f(x), g(x))$ ، ونقطة يمر بها منحنى $y = f(x)$.
أستعمل المعلومات المعطاة لإيجاد قاعدة الاقتران $(f(x), g(x))$:

(32) $(f(x), g(x)) = (2x(4x^2 - 10)^2, (2, 10))$

$$f(x) = \int f'(x) dx = \int 2x(4x^2 - 10)^2 dx \quad u = 4x^2 - 10 \Rightarrow \frac{du}{dx} = 8x \Rightarrow dx = \frac{du}{8x} \Rightarrow f(x) = \int 2x u^2 \frac{du}{8x} = \frac{1}{4} \int u^2 du = \frac{1}{12} u^3 + C \Rightarrow f(x) = \frac{1}{12} (4x^2 - 10)^3 + C$$

$$f(2) = \frac{1}{12} (216) + C = 10 \Rightarrow C = -8 \Rightarrow f(x) = \frac{1}{12} (4x^2 - 10)^3 - 8$$

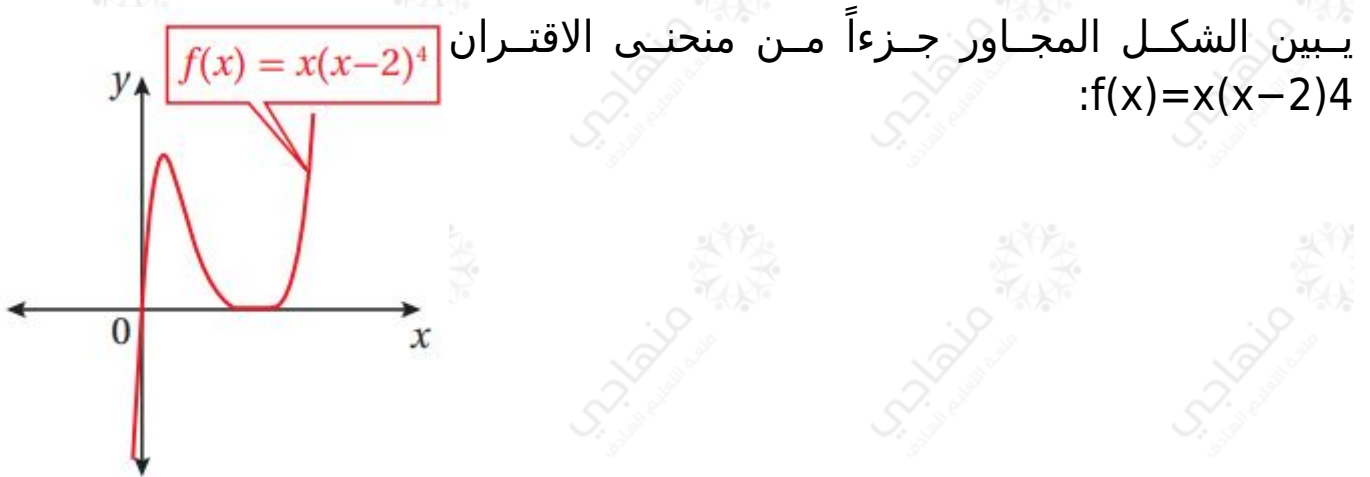
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$$(f'(x) = x^2 e^{-0.2x^3}; (0, 32)) \quad (33)$$

$$f(x) = \int f'(x) dx = \int x^2 e^{-0.2x^3} dx \quad u = -0.2x^3 \Rightarrow du/dx = -0.6x^2 \Rightarrow dx = du / -0.6x^2$$

$$x^2 f(x) = \int x^2 e^u du / -0.6x^2 = -1/0.6 \int e^u du = -5/3 e^u + C \Rightarrow f(x) = -5/3 e^{-0.2x^3} + C$$

$$+ C f(0) = -5/3 + C 32 = -5/3 + C \Rightarrow C = 196 \Rightarrow f(x) = -5/3 e^{-0.2x^3} + 196$$



(34) أجد إحداثي نقطة تماس الاقتران مع المحور x

نجد أصفار الاقتران بحل المعادلة $f(x) = 0$

$$x(x-2)^4 = 0 \Rightarrow x = 0, x = 2$$

نقطة التقاطع $(0, 0)$, فتكون نقطة التماس $(2, 0)$

ويمكن التحقق بحساب $f'(2)$:

$$f'(x) = (x-2)^4 + 4x(x-2)^3 \quad f'(2) = (2-2)^4 + 4(2)(2-2)^3 = 0$$

(35) أجد مساحة المنطقة المحصورة بين منحنى الاقتران $f(x)$ والمحور x

$$A = \int_0^2 x(x-2)^4 dx \quad u = x-2 \Rightarrow dx = du, x = u+2 \quad x=0 \Rightarrow u = -2 \quad x=2 \Rightarrow u = 0$$

$$A = \int_{-2}^0 (u+2)u^4 du = \int_{-2}^0 (u^5 + 2u^4) du = (1/6 u^6 + 2/5 u^5) \Big|_{-2}^0$$

$$= 0 - (1/6 (-2)^6 + 2/5 (-2)^5) = 32/15$$

(36) يتحرك جسيم في مسار مستقيم، وتعطى سرعته المتجهة بالاقتران:

$\omega t \cos 2v(t) = \sin$ حيث t الزمن بالثواني، و v سرعته المتجهة بالمتري لكل ثانية،

و b ثابت، إذا انطلق الجسم من نقطة الأصل، فأجد موقعه بعد t ثانية.

$$wts(t) = wt \Rightarrow dt = du - w \sin wt \Rightarrow dudx = -w \sin wt dt u = \cos wt \cos 2s(t) = f \sin wt + C wt = -1 w \int u^2 du = -13 w u^3 + C \Rightarrow s(t) = -130 \cos 3 w t u^2 du - w \sin f \sin$$

لكن $s(0) = 0$ لأن الجسم انطلق من نقطة الأصل.

$$wt + 13ws(0) = -13w + C0 = -13w + C \Rightarrow C = 13w \Rightarrow s(t) = -13w \cos 3$$



(37) طب: يمثل الاقتران $C(t)$ تركيز دواء في الدم بعد t دقيقة من حقنه في جسم مريض، حيث C مقيسة بالمليغرام لكل سنتيمتر مكعب (mg/cm^3)، إذا كان تركيز الدواء لحظة حقنه في جسم المريض $0.5 mg/cm^3$ ، وأخذ يتغير بمعدل $C'(t) = -0.01e^{-0.01t}(1+e^{-0.01t})^2$ ، فأجد $C(t)$.

$$C(t) = \int C'(t) dt = \int -0.01e^{-0.01t}(1+e^{-0.01t})^2 dt u = 1+e^{-0.01t} \Rightarrow dudt = -0.01e^{-0.01t} \Rightarrow dt = du - 0.01e^{-0.01t} C(t) = \int -0.01e^{-0.01t} u^2 \times du - 0.01e^{-0.01t} = \int u^2 du = -u^{-1} + K$$

استعمل الرمز K لثابت التكامل بدل C المعتاد لتمييز ثابت التكامل عن رمز الاقتران C :

$$C(t) = -(1+e^{-0.01t})^{-1} + K C(0) = -(2)^{-1} + K12 = -12 \Rightarrow K = 1 \Rightarrow C(t) = -(1+e^{-0.01t})^{-1} + 1 C(t) = -11 + e^{-0.01t} + 1$$

(38) أجد قيمة $\int \ln \ln x dx$ ، ثم اكتب الإجابة بالصيغة الآتية: $dab + c \ln$ ، حيث a, b, c, d ثوابت صحيحة.

$$3-2=3-2=1x=|3 \Rightarrow u = e \ln u = e x - 2 \Rightarrow dudx = e x \Rightarrow dx = du e x e x = u + 2x = \ln 4e4x e x - 2 dx = \int 12e4x u du e x = \int 12e3x u d3 \ln 4 - 2 = 4 - 2 = 2 \int \ln 4 \Rightarrow u = e \ln u = \int 12(u+2)3 u du = \int 12(u^2 + 6u + 12 + 8u) du |u|^{12} = (13u^3 + 3u^2 + 12u + 8 \ln$$

(39) إذا كان: $xf'(x) = \tan$ ، وكان: $f(3) = 5$ ، فأثبت أن $\ln |\cos x| + 53 \cos | \cos f(x) = \ln$.

$$3| + C5 = -\ln |\cos x| + C f(3) = -\ln |\cos x dx = -\ln x \cos x dx = -\int -\sin f(x) = \int \tan$$

$$x|+53\cos|\cos3|=\ln|\cos x|+5+\ln|\cos 3|f(x)=-\ln|\cos 3|+C\Rightarrow C=5+\ln|\cos$$