

إجابات كتاب التمارين

التكامل المحدود

أجد قيمة كل من التكاملات الآتية:

$$\int (1510x - 2) dx$$

$$1510x - 2 dx = -10x - 1 |_{15} = -10x |_{15} = (-2) - (-10) = 8$$

$$\int (2x^3 - 4x + 5) dx$$

$$2x^3 - 4x + 5 dx = (12x^4 - 2x^2 + 5x) |_{02} = 8 - 8 + 10 = 10$$

$$\int (14x^3 + 2x^2) dx$$

$$14x^3 + 2x^2 dx = \int 14(x^3 + 2x^2) dx = \int 14(x^5 + 2x^3) dx = (27x^7 + 45x^5) |_{14} = (2567 + 1285) - (27 + 45) = 2547 + 1245 = 213835$$

$$\int (x - 3x^2) dx$$

$$x - 3x^2 dx = \int 36(x^2 - 6 + 9x^2) dx = \int 36(x^2 - 6 + 9x - 2) dx = (13x^3 - 6x - 9x - 1) |_{36} = (13x^3 - 6x - 9x) |_{36} = (72 - 36 - 32) - (9 - 18 - 3) = 932$$

$$\int (x + 3) | -5 dx$$

$$x + 3 | -5 = \{-x - 3, x < -3; x + 3, x \geq -3\} \int 05(|x + 3| - 5) dx = \int 05(x + 3 - 5) dx = \int 05(x - 2) dx = (12x^2 - 2x) |_{05} = (252 - 10) - (0 - 0) = 52$$

$$\int 06x(6 - x) dx$$

$$06x(6 - x) dx = \int 06(6x - x^2) dx = (3x^2 - 13x^3) |_{06} = (108 - 2163) - (0) = 1083 - 36$$

$$\int (6x - 12x^4 + 3) dx$$

$$6x - 12x^4 + 3 dx = \int 12(6x - 12x - 4 + 3) dx = (3x^2 + 4x - 3 + 3x) |_{12} = (3) |_{12} = (12 + 12 + 6) - (3 + 4 + 3) = 172$$

$$\int_{-2}^2 (2x-1) dx \quad (8) \quad 07f$$

$$\begin{aligned} 2x-1 &= \begin{cases} -2x+1, & x < 1 \\ 2x-1, & x \geq 1 \end{cases} \\ \int_{-2}^2 (2x-1) dx &= \int_{-2}^1 (-2x+1) dx + \int_1^2 (2x-1) dx \\ &= (-x^2+x) \Big|_{-2}^1 + (x^2-x) \Big|_1^2 = (-14+12) - (0) + (49-7) - (14-12) = 852 \end{aligned}$$

$$\int_{-3}^4 x dx \quad (9) \quad 34-f$$

$$\begin{aligned} x &= \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases} \\ \int_{-3}^4 x dx &= \int_{-3}^0 -x dx + \int_0^4 x dx = -12x^2 \Big|_{-3}^0 - 30 + 12x^2 \Big|_0^4 \\ &= (0) - (-92) + (8) - (0) = 252 \end{aligned}$$

$$\int_{-12}^{12} (2x^2+x^3) dx \quad (10) f$$

$$\begin{aligned} \int_{-12}^{12} (2x^2+x^3) dx &= \int_{-12}^{12} (2x^2+x^3) dx = \int_{-12}^{12} (2x^2+x^3) dx \\ &= (2x^3 + \frac{1}{4}x^4) \Big|_{-12}^{12} = (2 \cdot 12^3 + \frac{1}{4} \cdot 12^4) - (2 \cdot (-12)^3 + \frac{1}{4} \cdot (-12)^4) \\ &= (288 + 32) - (-288 + 32) = 32 + 73 = 236 \end{aligned}$$

$$\int_{-34}^4 (6x^2-4x) dx \quad (11) \quad 34f$$

$$\int_{-34}^4 (6x^2-4x) dx = (2x^3+2x^2) \Big|_{-34}^4 = (128+32) - (54+18) = 88 \quad 34f$$

$$\int_{-10}^0 (10x+1x^2) dx \quad (12) f$$

$$\int_{-10}^0 (10x+1x^2) dx = 0 f$$

إذا كان $\int_{-2}^6 g(x) dx = -2$, $\int_{-3}^2 f(x) dx = 4$, $\int_{-3}^2 f(x) dx = 5$, فأجد كلاً مما يأتي:

$$\int_{-2}^2 2f(x) dx \quad (13) f$$

$$\int_{-2}^2 2f(x) dx = 0 f$$

$$\int_{-12}^{12} (f(x)-5) dx \quad (14) \quad 12f$$

$$\begin{aligned} \int_{-12}^{12} (f(x)-5) dx &= \int_{-12}^{12} f(x) dx - \int_{-12}^{12} 5 dx = \int_{-1}^1 3f(x) dx + \int_{-3}^2 f(x) dx + \int_{-5}^{12} (12-5) dx \\ &= -4 + 5 + (-5x) \Big|_{-12}^{12} = 1 + (-10) - (-5) = -4 \end{aligned}$$

$$\int_{-32}^{-15} (2f(x)+5g(x)) dx \quad (15) \quad -f$$

$$\int_{-32}^{-15} (2f(x)+5g(x)) dx = -2 \int_{-3}^2 f(x) dx + 5 \int_{-2}^6 g(x) dx = -2(5) + 5(-2) = -32 - f$$

$$=-20$$

$$\int (g(x)+2x)dx \quad (16) \quad 2-3 \int$$

$$\int (g(x)+2x)dx = \int 2-3g(x)dx + \int 2-32xdx = -(-2) + (x^2)|_{2-3} = 2+9-(-)2-3 \int$$

$$4=7$$

$$\int (f(x)+g(x))dx \quad (17) \quad 2-3 \int$$

$$\int (f(x)+g(x))dx = \int 2-3f(x)dx + \int 2-3g(x)dx = -5+2 = -3)2-3 \int$$

$$\int (4f(x)-3g(x))dx \quad (18) \quad 32-\int$$

$$\int (4f(x)-3g(x))dx = 4\int -32f(x)dx - 3\int -32g(x)dx = 4(5) - 3(-2) = 26)32-\int$$

(19) إذا كان $f(x) = \begin{cases} x^2, & x < 2 \\ 28-x, & x \geq 2 \end{cases}$ ، فأجد قيمة $\int -36f(x)dx$.

$$\int -36f(x)dx = \int -32f(x)dx + \int 26f(x)dx = \int -32x^2dx + \int 26(8-x)dx = (13x^3) - \int$$

$$|-32 + (8x - 12x^2)|_{26} = (83) - (-9) + (48 - 18) - (16 - 2) = 833$$

(20) سكان: أشارت دراسة إلى أن عدد السكان في إحدى القرى يتغير شهرياً بمعدل يمكن نمذجته بالاقتران: $P'(t) = 5 + 3t^{2/3}$ ، حيث t عدد الأشهر من الآن، و $P(t)$ عدد السكان، أجد مقدار الزيادة في عدد سكان القرية في الأشهر الثمانية القادمة.

$$P(t) = \int_0^8 (5 + 3t^{2/3})dt = (5t + 95t^{5/3})|_0^8 = (40 + 2885) - (0) = 4885$$

(21) إذا كان: $\int (x^2 - a)dx = 5)23 \int$ ، فأجد قيمة الثابت a .

$$\int (x^2 - a)dx = 5(13x^3 - ax)|_{23} = 5(9 - 3a) - (83 - 2a) = 5173 - \alpha = 5a = 23)23 \int$$