

## أدرب وأحل المسائل

### التكامل بالتعويض

أجد كلاً من التكاملات الآتية:

$$\int (x^2+4) dx \quad (1)$$

$$\begin{aligned} x^2+4 dx u=x^2+4 \Rightarrow du dx=2x \Rightarrow dx=du/2x \\ \int (x^2+4) dx = \int x u \times du/2x = \int 1/2 u \\ du = \int 1/2 u - 1/2 du = u^2/2 + C = x^2+4 + C \end{aligned}$$

$$\int x^2(2x^3+5)^4 dx \quad (2)$$

$$\begin{aligned} x^2(2x^3+5)^4 dx u=2x^3+5 \Rightarrow du dx=6x^2 \Rightarrow dx=du/6x^2 \\ \int x^2(2x^3+5)^4 dx = \int x^2/6x^2 \int u^4 \\ du = \int 1/6 u^4 du = 1/30 u^5 + C = 1/30(2x^3+5)^5 + C \end{aligned}$$

$$\int (3x^2+7) dx \quad (3)$$

$$\begin{aligned} 3x^2+7 dx u=x^2+7 \Rightarrow du dx=2x \Rightarrow dx=du/2x \\ \int (3x^2+7) dx = \int 3x u \times du/2x = \int 3/2 u \\ du = u^3/2 + C = (x^2+7)^3/2 + C \end{aligned}$$

$$\int x^6 e^{1-x^7} dx \quad (4)$$

$$\begin{aligned} x^6 e^{1-x^7} dx u=1-x^7 \Rightarrow du dx=-7x^6 \Rightarrow dx=du/-7x^6 \\ \int x^6 e^{1-x^7} dx = \int x^6 e^u \times du/-7x^6 \\ = \int -1/7 e^u du = -1/7 e^u + C = -1/7 e^{1-x^7} + C \end{aligned}$$

$$\int x^4(x^5+9)^3 dx \quad (5)$$

$$\begin{aligned} x^4(x^5+9)^3 dx u=x^5+9 \Rightarrow du dx=5x^4 \Rightarrow dx=du/5x^4 \\ \int x^4(x^5+9)^3 dx = \int x^4 u^3 \times du/5x^4 \\ = \int 1/5 u^3 du = 1/20 u^4 + C = 1/20(x^5+9)^4 + C \end{aligned}$$

$$\int (3x^2-1) e^{x^3-x} dx \quad (6)$$

$$\begin{aligned} (3x^2-1) e^{x^3-x} dx u=x^3-x \Rightarrow du dx=3x^2-1 \Rightarrow dx=du/(3x^2-1) \\ \int (3x^2-1) e^{x^3-x} dx = \int (3x^2-1) e^u \times du/(3x^2-1) \\ = \int e^u du = e^u + C = e^{x^3-x} + C \end{aligned}$$

$$\int (3x-3x^2-2x+4) dx \quad (7)$$

$$\int (3x-3x^2-2x+4) dx u=x^2-2x+4 \Rightarrow du dx=2x-2 \Rightarrow dx=du/(2x-2)$$

$$x^2 - 2x + 4 dx = \int 3x - 3u \times du \quad 2x - 2 = \int 3(x-1)u \times du \quad 2(x-1) = \int 32u - 12 du = 3u^2 + C = 3x^2 - 2x + 4 + C$$

$$(x dx) \quad (81x \ln f$$

$$|x dx = \int 1xu \times x du = \int 1u du = \ln x \Rightarrow du dx = 1x \Rightarrow dx = x du \int 1x \ln x dx = \ln |1x \ln x| + C \quad |\ln u| + C = \ln$$

$$(x)4 dx \quad (9x(1 + \cos \sin f$$

$$x(1 + \cos x) \int \sin x \Rightarrow dx = du - \sin x \Rightarrow du dx = -\sin x \quad 4 dx u = 1 + \cos x (1 + \cos \sin f x)5 + C x = \int -u^4 du = -15u^5 + C = -15(1 + \cos x)u^4 \times du - \sin x)4 dx = \int \sin$$

$$(2x dx) \quad (102x \cos \sin 5 f$$

$$2x \cos 2x \int \sin 5 2x \Rightarrow dx = du \quad 2 \cos 2x \Rightarrow du dx = 2 \cos 2x \quad dx u = \sin 2x \cos \sin 5 2x)6 + C 2x = \int 12u^5 du = 112u^6 + C = 112(\sin 2x \times du \quad 2 \cos 2x dx = \int u^5 \cos$$

$$((1x)x^2 dx) \quad (11 \sin f$$

$$(u)x^2 (1x)x^2 dx = \int \sin(1x)x^2 dx u = 1x \Rightarrow du dx = -1x^2 \Rightarrow dx = -x^2 du \int \sin \sin f (1x) + C u + C = \cos u du = \cos x - x^2 du = \int -\sin$$

$$(x dx) \quad (12x e \sin \cos f$$

$$x e dx = \int \cos x e \sin x \int \cos x \Rightarrow dx = du \quad \cos x \Rightarrow du dx = \cos x \quad dx u = \sin x e \sin \cos f x + C x + C = -1 e \sin x = \int 1 e u du = \int e^{-u} du = -e^{-u} + C = -e^{-\sin x} \times du \cos$$

$$(e x(2 + e x)5 dx) \quad (13 f$$

$$e x(2 + e x)5 dx u = 2 + e x \Rightarrow du dx = e x \Rightarrow dx = du \quad e x \int e x(2 + e x)5 dx = \int e x u^5 \times d f u e x = \int u^5 du = 16u^6 + C = 16(2 + e x)^6 + C$$

$$(x) x dx \quad (14(\ln \cos f$$

$$(u)x \times x du = x) x dx = \int \cos(\ln x \Rightarrow du dx = 1x \Rightarrow dx = x du \int \cos x) x dx u = \ln(\ln \cos f x) + C(\ln u + C = \sin u du = \sin \int \cos$$

$$(3x^2 - 2x - 1)(x^3 - x^2 - x)4 dx \quad (15) f$$

$$\int (3x^2 - 2x - 1)(x^3 - x^2 - x)^4 dx \quad u = x^3 - x^2 - x \Rightarrow \frac{du}{dx} = 3x^2 - 2x - 1 \Rightarrow dx = \frac{du}{3x^2 - 2x - 1}$$

$$\int (3x^2 - 2x - 1) (3x^2 - 2x - 1)(x^3 - x^2 - x)^4 dx = \int (3x^2 - 2x - 1) u^4 \times \frac{du}{3x^2 - 2x - 1}$$

$$= \int u^4 du = \frac{1}{5} u^5 + C = \frac{1}{5} (x^3 - x^2 - x)^5 + C$$

أجد قيمة كل من التكاملات الآتية:

(16)  $\int (2x-1)ex^{2-x} dx$

$$u = x^2 - x \Rightarrow \frac{du}{dx} = 2x - 1 \Rightarrow dx = \frac{du}{2x - 1}$$

$$\int (2x - 1) e^{x^2 - x} dx = \int (2x - 1) e^u \times \frac{du}{2x - 1}$$

$$= \int e^u du = e^u + C = e^{x^2 - x} + C$$

(17)  $\int \frac{1}{x^2} e^{1/x} dx$

$$u = \frac{1}{x} \Rightarrow \frac{du}{dx} = -\frac{1}{x^2} \Rightarrow dx = -x^2 du$$

$$\int \frac{1}{x^2} e^{1/x} dx = \int e^u \times (-x^2 du) = -\int e^u du = -e^u + C = -e^{1/x} + C$$

(18)  $\int x e^{3 \ln x} dx$

$$u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x} \Rightarrow dx = x du$$

$$\int x e^{3 \ln x} dx = \int e^{3u} \times x du = \int e^{3u} \times e^{3u} du = \int e^{6u} du = \frac{1}{6} e^{6u} + C = \frac{1}{6} e^{6 \ln x} + C = \frac{1}{6} x^6 + C$$

(19)  $\int (x^3 + x)x^4 + 2x^2 + 1 dx$

$$u = x^4 + 2x^2 + 1 \Rightarrow \frac{du}{dx} = 4x^3 + 4x \Rightarrow dx = \frac{du}{4x^3 + 4x}$$

$$\int (x^3 + x) (x^4 + 2x^2 + 1) dx = \int (x^3 + x) u \times \frac{du}{4x^3 + 4x}$$

$$= \int \frac{(x^3 + x) u du}{4(x^3 + x)} = \frac{1}{4} \int u du = \frac{1}{4} \times \frac{1}{2} u^2 + C = \frac{1}{8} (x^4 + 2x^2 + 1)^2 + C$$

(20)  $\int (3x^2 + 1) dx$

$$u = x^2 + 1 \Rightarrow \frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$$

$$\int (3x^2 + 1) dx = \int (3x^2 + 1) \times \frac{du}{2x} = \int \frac{3x^2 + 1}{2x} du = \int \frac{3x^2 + 1}{2x} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{4x^2} du = \int \frac{3x^2 + 1}{4x^2} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{8x^3} du = \int \frac{3x^2 + 1}{8x^3} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{16x^4} du = \int \frac{3x^2 + 1}{16x^4} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{32x^5} du = \int \frac{3x^2 + 1}{32x^5} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{64x^6} du = \int \frac{3x^2 + 1}{64x^6} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{128x^7} du = \int \frac{3x^2 + 1}{128x^7} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{256x^8} du = \int \frac{3x^2 + 1}{256x^8} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{512x^9} du = \int \frac{3x^2 + 1}{512x^9} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{1024x^{10}} du = \int \frac{3x^2 + 1}{1024x^{10}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{2048x^{11}} du = \int \frac{3x^2 + 1}{2048x^{11}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{4096x^{12}} du = \int \frac{3x^2 + 1}{4096x^{12}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{8192x^{13}} du = \int \frac{3x^2 + 1}{8192x^{13}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{16384x^{14}} du = \int \frac{3x^2 + 1}{16384x^{14}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{32768x^{15}} du = \int \frac{3x^2 + 1}{32768x^{15}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{65536x^{16}} du = \int \frac{3x^2 + 1}{65536x^{16}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{131072x^{17}} du = \int \frac{3x^2 + 1}{131072x^{17}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{262144x^{18}} du = \int \frac{3x^2 + 1}{262144x^{18}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{524288x^{19}} du = \int \frac{3x^2 + 1}{524288x^{19}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{1048576x^{20}} du = \int \frac{3x^2 + 1}{1048576x^{20}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{2097152x^{21}} du = \int \frac{3x^2 + 1}{2097152x^{21}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{4194304x^{22}} du = \int \frac{3x^2 + 1}{4194304x^{22}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{8388608x^{23}} du = \int \frac{3x^2 + 1}{8388608x^{23}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{16777216x^{24}} du = \int \frac{3x^2 + 1}{16777216x^{24}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{33554432x^{25}} du = \int \frac{3x^2 + 1}{33554432x^{25}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{67108864x^{26}} du = \int \frac{3x^2 + 1}{67108864x^{26}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{134217728x^{27}} du = \int \frac{3x^2 + 1}{134217728x^{27}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{268435456x^{28}} du = \int \frac{3x^2 + 1}{268435456x^{28}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{536870912x^{29}} du = \int \frac{3x^2 + 1}{536870912x^{29}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{1073741824x^{30}} du = \int \frac{3x^2 + 1}{1073741824x^{30}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{2147483648x^{31}} du = \int \frac{3x^2 + 1}{2147483648x^{31}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{4294967296x^{32}} du = \int \frac{3x^2 + 1}{4294967296x^{32}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{8589934592x^{33}} du = \int \frac{3x^2 + 1}{8589934592x^{33}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{17179869184x^{34}} du = \int \frac{3x^2 + 1}{17179869184x^{34}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{34359738368x^{35}} du = \int \frac{3x^2 + 1}{34359738368x^{35}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{68719476736x^{36}} du = \int \frac{3x^2 + 1}{68719476736x^{36}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{137438953472x^{37}} du = \int \frac{3x^2 + 1}{137438953472x^{37}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{274877906944x^{38}} du = \int \frac{3x^2 + 1}{274877906944x^{38}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{549755813888x^{39}} du = \int \frac{3x^2 + 1}{549755813888x^{39}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{1099511627776x^{40}} du = \int \frac{3x^2 + 1}{1099511627776x^{40}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{2199023255552x^{41}} du = \int \frac{3x^2 + 1}{2199023255552x^{41}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{4398046511104x^{42}} du = \int \frac{3x^2 + 1}{4398046511104x^{42}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{8796093022208x^{43}} du = \int \frac{3x^2 + 1}{8796093022208x^{43}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{17592186044416x^{44}} du = \int \frac{3x^2 + 1}{17592186044416x^{44}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{35184372088832x^{45}} du = \int \frac{3x^2 + 1}{35184372088832x^{45}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{70368744177664x^{46}} du = \int \frac{3x^2 + 1}{70368744177664x^{46}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{140737488355328x^{47}} du = \int \frac{3x^2 + 1}{140737488355328x^{47}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{281474976710656x^{48}} du = \int \frac{3x^2 + 1}{281474976710656x^{48}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{562949953421312x^{49}} du = \int \frac{3x^2 + 1}{562949953421312x^{49}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{1125899906842624x^{50}} du = \int \frac{3x^2 + 1}{1125899906842624x^{50}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{2251799813685248x^{51}} du = \int \frac{3x^2 + 1}{2251799813685248x^{51}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{4503599627370496x^{52}} du = \int \frac{3x^2 + 1}{4503599627370496x^{52}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{9007199254740992x^{53}} du = \int \frac{3x^2 + 1}{9007199254740992x^{53}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{18014398509481984x^{54}} du = \int \frac{3x^2 + 1}{18014398509481984x^{54}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{36028797018963968x^{55}} du = \int \frac{3x^2 + 1}{36028797018963968x^{55}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{72057594037927936x^{56}} du = \int \frac{3x^2 + 1}{72057594037927936x^{56}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{144115188075855872x^{57}} du = \int \frac{3x^2 + 1}{144115188075855872x^{57}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{288230376151711744x^{58}} du = \int \frac{3x^2 + 1}{288230376151711744x^{58}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{576460752303423488x^{59}} du = \int \frac{3x^2 + 1}{576460752303423488x^{59}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{1152921504606846976x^{60}} du = \int \frac{3x^2 + 1}{1152921504606846976x^{60}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{2305843009213693952x^{61}} du = \int \frac{3x^2 + 1}{2305843009213693952x^{61}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{4611686018427387904x^{62}} du = \int \frac{3x^2 + 1}{4611686018427387904x^{62}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{9223372036854775808x^{63}} du = \int \frac{3x^2 + 1}{9223372036854775808x^{63}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{18446744073709551616x^{64}} du = \int \frac{3x^2 + 1}{18446744073709551616x^{64}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{36893488147419103232x^{65}} du = \int \frac{3x^2 + 1}{36893488147419103232x^{65}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{73786976294838206464x^{66}} du = \int \frac{3x^2 + 1}{73786976294838206464x^{66}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{147573952589676412928x^{67}} du = \int \frac{3x^2 + 1}{147573952589676412928x^{67}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{295147905179352825856x^{68}} du = \int \frac{3x^2 + 1}{295147905179352825856x^{68}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{590295810358705651712x^{69}} du = \int \frac{3x^2 + 1}{590295810358705651712x^{69}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{1180591620717411303424x^{70}} du = \int \frac{3x^2 + 1}{1180591620717411303424x^{70}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{2361183241434822606848x^{71}} du = \int \frac{3x^2 + 1}{2361183241434822606848x^{71}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{4722366482869645213696x^{72}} du = \int \frac{3x^2 + 1}{4722366482869645213696x^{72}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{9444732965739290427392x^{73}} du = \int \frac{3x^2 + 1}{9444732965739290427392x^{73}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{18889465931478580854784x^{74}} du = \int \frac{3x^2 + 1}{18889465931478580854784x^{74}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{37778931862957161709568x^{75}} du = \int \frac{3x^2 + 1}{37778931862957161709568x^{75}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{75557863725914323419136x^{76}} du = \int \frac{3x^2 + 1}{75557863725914323419136x^{76}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{151115727451828646838272x^{77}} du = \int \frac{3x^2 + 1}{151115727451828646838272x^{77}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{302231454903657293676544x^{78}} du = \int \frac{3x^2 + 1}{302231454903657293676544x^{78}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{604462909807314587353088x^{79}} du = \int \frac{3x^2 + 1}{604462909807314587353088x^{79}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{1208925819614629174706176x^{80}} du = \int \frac{3x^2 + 1}{1208925819614629174706176x^{80}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{2417851639229258349412352x^{81}} du = \int \frac{3x^2 + 1}{2417851639229258349412352x^{81}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{4835703278458516698824704x^{82}} du = \int \frac{3x^2 + 1}{4835703278458516698824704x^{82}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{9671406556917033397649408x^{83}} du = \int \frac{3x^2 + 1}{9671406556917033397649408x^{83}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{19342813113834066795298816x^{84}} du = \int \frac{3x^2 + 1}{19342813113834066795298816x^{84}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{38685626227668133590597632x^{85}} du = \int \frac{3x^2 + 1}{38685626227668133590597632x^{85}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{77371252455336267181195264x^{86}} du = \int \frac{3x^2 + 1}{77371252455336267181195264x^{86}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{154742504910672534362390528x^{87}} du = \int \frac{3x^2 + 1}{154742504910672534362390528x^{87}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{309485009821345068724781056x^{88}} du = \int \frac{3x^2 + 1}{309485009821345068724781056x^{88}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{618970019642690137449562112x^{89}} du = \int \frac{3x^2 + 1}{618970019642690137449562112x^{89}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{1237940039285380274899124224x^{90}} du = \int \frac{3x^2 + 1}{1237940039285380274899124224x^{90}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{2475880078570760549798248448x^{91}} du = \int \frac{3x^2 + 1}{2475880078570760549798248448x^{91}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{4951760157141521099596496896x^{92}} du = \int \frac{3x^2 + 1}{4951760157141521099596496896x^{92}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{9903520314283042199192993792x^{93}} du = \int \frac{3x^2 + 1}{9903520314283042199192993792x^{93}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{19807040628566084398385987584x^{94}} du = \int \frac{3x^2 + 1}{19807040628566084398385987584x^{94}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{39614081257132168796771975168x^{95}} du = \int \frac{3x^2 + 1}{39614081257132168796771975168x^{95}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{79228162514264337593543950336x^{96}} du = \int \frac{3x^2 + 1}{79228162514264337593543950336x^{96}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{158456325028528675187087900672x^{97}} du = \int \frac{3x^2 + 1}{158456325028528675187087900672x^{97}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{316912650057057350374175801344x^{98}} du = \int \frac{3x^2 + 1}{316912650057057350374175801344x^{98}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{633825300114114700748351602688x^{99}} du = \int \frac{3x^2 + 1}{633825300114114700748351602688x^{99}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{1267650600228229401496703205376x^{100}} du = \int \frac{3x^2 + 1}{1267650600228229401496703205376x^{100}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{2535301200456458802993406410752x^{101}} du = \int \frac{3x^2 + 1}{2535301200456458802993406410752x^{101}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{5070602400912917605986812821504x^{102}} du = \int \frac{3x^2 + 1}{5070602400912917605986812821504x^{102}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{10141204801825835211973625643008x^{103}} du = \int \frac{3x^2 + 1}{10141204801825835211973625643008x^{103}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{20282409603651670423947251286016x^{104}} du = \int \frac{3x^2 + 1}{20282409603651670423947251286016x^{104}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{40564819207303340847894502572032x^{105}} du = \int \frac{3x^2 + 1}{40564819207303340847894502572032x^{105}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{81129638414606681695789005144064x^{106}} du = \int \frac{3x^2 + 1}{81129638414606681695789005144064x^{106}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{162259276829213363391578010288128x^{107}} du = \int \frac{3x^2 + 1}{162259276829213363391578010288128x^{107}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{324518553658426726783156020576256x^{108}} du = \int \frac{3x^2 + 1}{324518553658426726783156020576256x^{108}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{649037107316853453566312041152512x^{109}} du = \int \frac{3x^2 + 1}{649037107316853453566312041152512x^{109}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{1298074214633706907132624082305024x^{110}} du = \int \frac{3x^2 + 1}{1298074214633706907132624082305024x^{110}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{2596148429267413814265248164610048x^{111}} du = \int \frac{3x^2 + 1}{2596148429267413814265248164610048x^{111}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{5192296858534827628530496329220096x^{112}} du = \int \frac{3x^2 + 1}{5192296858534827628530496329220096x^{112}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{10384593717069655257060992658440192x^{113}} du = \int \frac{3x^2 + 1}{10384593717069655257060992658440192x^{113}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{20769187434139310514121985316880384x^{114}} du = \int \frac{3x^2 + 1}{20769187434139310514121985316880384x^{114}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{41538374868278621028243970633760768x^{115}} du = \int \frac{3x^2 + 1}{41538374868278621028243970633760768x^{115}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{83076749736557242056487941267521536x^{116}} du = \int \frac{3x^2 + 1}{83076749736557242056487941267521536x^{116}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{166153499473114484112975882535043072x^{117}} du = \int \frac{3x^2 + 1}{166153499473114484112975882535043072x^{117}} \times \frac{du}{2x}$$

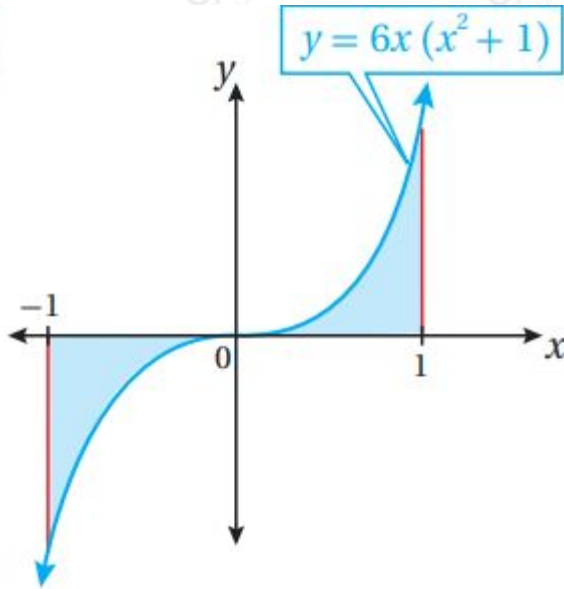
$$= \int \frac{3x^2 + 1}{332306998946228968225951765070086144x^{118}} du = \int \frac{3x^2 + 1}{332306998946228968225951765070086144x^{118}} \times \frac{du}{2x} = \int \frac{3x^2 + 1}{664613997892457936451903530140172288x^{119}} du = \int \frac{3x^2 + 1}{664613997892457936451903530140172288x^{119}} \times \frac{du}{2x}$$

$$= \int \frac{3x^2 + 1}{13292279957849$$

$$= (2)^2 + 2 + 4 = 10 \quad x=1 \Rightarrow u=(1)^2+1+4=6 \quad \int_{-1}^1 2x+1(x^2+x+4)^3 dx = \int_{-1}^1 6 \cdot 10^2 x+1 u^3 \times du \quad 2x+1 = \int_{6 \cdot 10}^6 10 u - 3 du = -12u - 2 \Big|_{6 \cdot 10}^6 = -12u^2 \Big|_{6 \cdot 10}$$

أجد مساحة المنطقة المظللة في كل من التمثيلين البيانيين الآتيين:

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$$A = - \int_{-1}^0 6x(x^2+1) dx + \int_0^1 6x(x^2+1) dx$$

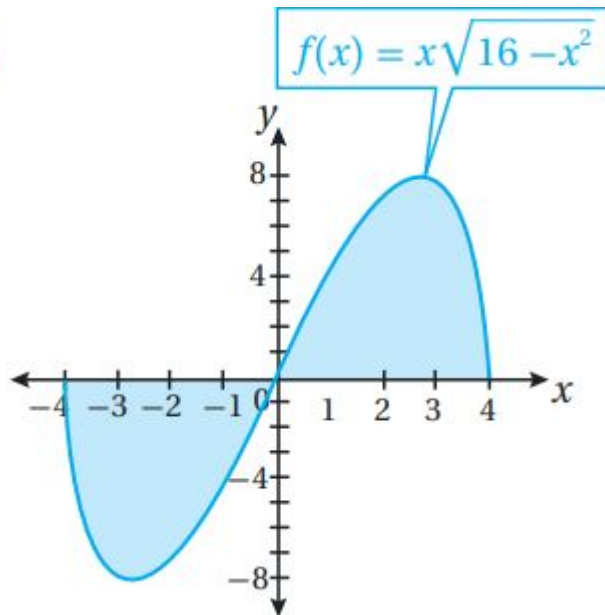
هناك طريقتان للحل: إما التكامل بالتعويض، أو تكامل كثير حدود بعد توزيع الأقواس.

طريقة التكامل بالتعويض:

$$u = x^2 + 1 \Rightarrow \frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x} \quad x=0 \Rightarrow u=(0)^2+1=1 \quad x=-1 \Rightarrow u=(-1)^2+1=2 \quad x=1 \Rightarrow u=(1)^2+1=2 \quad A = - \int_2^1 6x(x^2+1) \frac{du}{2x} + \int_1^2 6x(x^2+1) \frac{du}{2x} = - \int_2^1 3u du + \int_1^2 3u du = - \frac{3}{2} u^2 \Big|_2^1 + \frac{3}{2} u^2 \Big|_1^2 = - \frac{3}{2} (1)^2 + \frac{3}{2} (2)^2 + \frac{3}{2} (2)^2 - \frac{3}{2} (1)^2 = 9$$

ومنه مساحة المنطقة المظللة هي 9 وحدات مربعة.

23



$$A = -\int_{-4}^0 x\sqrt{16-x^2} dx + \int_0^4 x\sqrt{16-x^2} dx$$

$$u = 16 - x^2 \Rightarrow \frac{du}{dx} = -2x \Rightarrow dx = \frac{du}{-2x}$$

$$2xx = 0 \Rightarrow u = 16 - (0)^2 = 16 \quad x = -4 \Rightarrow u = 16 - (-4)^2 = 0$$

$$x = 4 \Rightarrow u = 16 - (4)^2 = 0$$

$$A = -\int_{16}^0 \frac{1}{2} \sqrt{u} \frac{du}{-2x} + \int_0^4 x\sqrt{16-x^2} dx = -\int_0^{16} \frac{1}{2} \sqrt{u} \frac{du}{-2x} + \int_0^4 x\sqrt{16-x^2} dx$$

$$= \int_0^{16} \frac{1}{4} \sqrt{u} du + \int_0^4 x\sqrt{16-x^2} dx = \frac{1}{4} \cdot \frac{2}{3} u^{3/2} \Big|_0^{16} + \int_0^4 x\sqrt{16-x^2} dx$$

$$= \frac{1}{6} (16)^{3/2} - \frac{1}{6} (0)^{3/2} + \int_0^4 x\sqrt{16-x^2} dx = \frac{1}{6} (64) - 0 + \int_0^4 x\sqrt{16-x^2} dx = \frac{8}{3} + \int_0^4 x\sqrt{16-x^2} dx$$

ومنه مساحة المنطقة المظللة هي 1283 وحدات مربعة.

في كل مما يأتي المشتقة الأولى للاقتران  $f(x)$ ، ونقطة يمر بها منحنى  $y=f(x)$  أستعمل المعلومات المعطاة لإيجاد قاعدة الاقتران  $f(x)$ :

(24)  $f'(x) = xe^{4-x^2}; (-2, 1)$

$$f(x) = \int xe^{4-x^2} dx \quad u = 4 - x^2 \Rightarrow \frac{du}{dx} = -2x \Rightarrow dx = \frac{du}{-2x}$$

$$f(x) = \int xe^{4-x^2} dx = \int \frac{1}{2} e^u \frac{du}{-2x} = -\frac{1}{4} \int e^u du = -\frac{1}{4} e^u + C = -\frac{1}{4} e^{4-x^2} + C$$

لإيجاد ثابت التكامل، نعوض النقطة  $(-2, 1)$ :

$$f(x) = -\frac{1}{4} e^{4-x^2} + C \Rightarrow f(-2) = -\frac{1}{4} e^{4-(-2)^2} + C \Rightarrow 1 = -\frac{1}{4} e^0 + C \Rightarrow C = \frac{5}{4}$$

$$f(x) = -\frac{1}{4} e^{4-x^2} + \frac{5}{4}$$

(25)  $f'(x) = 2x(1-x^2)^2; (0, -1)$

$$f(x) = \int 2x(1-x^2)^2 dx \quad u = 1 - x^2 \Rightarrow \frac{du}{dx} = -2x \Rightarrow dx = \frac{du}{-2x}$$

$$f(x) = \int 2x(1-x^2)^2 dx = \int -1(1-u)^2 du = -\int (1-u)^2 du = -\int (1-2u+u^2) du = -\left(u - u^2 + \frac{1}{3}u^3\right) + C$$

$$= -\left(1-x^2 - (1-x^2)^2 + \frac{1}{3}(1-x^2)^3\right) + C$$

$$= \int 2xu^2 \times du - 2x = \int -u - 2du = u - 1 + C = 11 - x^2 + C$$

لإيجاد ثابت التكامل، نعوض النقطة 0، -1:

$$f(x) = 11 - x^2 + C \Rightarrow f(0) = 11 - 0^2 + C \Rightarrow -1 = 1 + C \Rightarrow C = -2$$

(26) يتحرك جسيم في مسار مستقيم، وتعطى سرعته المتجهة بالاقتران:  
 $v(t) = -2t(1+t^2)^3$ ، حيث  $t$  الزمن بالثواني، و  $v$  سرعته المتجهة بالمتري لكل ثانية. إذا  
 كان الموقع الابتدائي للجسيم  $m$  4، فأجد موقع الجسيم بعد  $t$  ثانية من بدء الحركة.

$$s(t) = \int -2t(1+t^2)^3 dt \quad u = 1+t^2 \Rightarrow du/dt = 2t \Rightarrow dt = du/2t$$

$$\int -2t(1+t^2)^3 dt = \int -2t \times du/2t = \int -du = -u + C = -2(1+t^2) + C = -2 - 2t^2 + C$$

بما أن الموقع الابتدائي للجسيم  $m$  4، إذن،  $s(0) = 4$ :

$$s(t) = -2 - 2t^2 + C \Rightarrow f(0) = -2 - 2(0)^2 + C \Rightarrow 4 = -2 + C \Rightarrow C = 6$$



(27) زراعة: يمثل الاقتران  $V(t)$  سعر دونم أرض  
 زراعية في الأغوار الأردنية (بالدينار) بعد  $t$  سنة من  
 الآن. إذا كان:  $V'(t) = 0.4t^3 - 0.2t^4 + 8000$  هو  
 معدل التغير في سعر دونم الأرض، فأجد  $V(t)$ ، علماً  
 بأن سعره الآن 5000 JD.

$$V(t) = \int (0.4t^3 - 0.2t^4 + 8000) dt = 0.1t^4 - 0.04t^5 + 8000t + C$$

$$V(t) = \int (0.4t^3 - 0.2t^4 + 8000) dx = \int 0.4t^3 u^3 \times du/0.8t^3 = \int 1/2 u^3 du = 1/8 u^4 + C$$

$$= 1/8 (0.2t^4 + 8000)^4 + C$$

بما أن سعر دونم الأرض الآن هو 5000 دينار، إذن،  $V(0) = 5000$  ومنه:

$$V(t) = 1/8 (0.2t^4 + 8000)^4 + C \quad V(0) = 1/8 (0.2(0)^4 + 8000)^4 + C = 5000$$

$$1/8 (8000)^4 + C = 5000 \Rightarrow C = 5000 - 1/8 (8000)^4$$

$$V(t) = 1/8 (0.2t^4 + 8000)^4 + 5000 - 1/8 (8000)^4$$

(28) سكان: أشارت دراسة إلى أن عدد السكان في إحدى المدن يتغير سنوياً بمعدل  
 يمكن نمذجته بالاقتران:  $P'(t) = 4e^{0.2t^4} + e^{0.2t}$ ، حيث  $t$  عدد السنوات منذ عام

2015 م، و  $P(t)$  عدد السكان بالآلاف. أجد مقدار الزيادة في عدد السكان عام 2015 م إلى عام 2025 م.

$$\begin{aligned} dt &= du \quad 0.2e^{0.2t} = 10 \Rightarrow u = 4 + e^{0.2(10)} = 4 + e^2 \\ t = 0 &\Rightarrow u = 4 + e^{0.2(0)} = 5 \\ \int_0^{10} 0.2e^{0.2t} dt &= \int_5^{4+e^2} du \\ 10 &= 4 + e^2 - 5 \\ e^2 &= 1 \\ e &= 1 \\ 2 &= 0 \end{aligned}$$

إذن يزداد عدد سكان هذه المدينة بحوالي 46 ألف شخص من 2015 م إلى 2025 م.